A STUDY ON THE FREQUENCY AND DAMPING OF SOIL - STRUCTURE SYSTEMS USING A SIMPLIFIED MODEL

*M. Ali GHANNAD**, *Nobuo FUKUWA*** *and Riei NISHIZAKA****

A simplified 3DOF model is suggested as the soil-structure system by replacing the structure by a SDOF model using the modal coordinates of the fixed-base structure. The modal parameters of the system are evaluated parametrically through the eigenvalue analysis. The derived formulations allow the use of hysteretic damping model as the material damping in the soil and structure which is a more realistic model than the commonly used viscous one. The frequency dependency of the soil impedances can also be handled by the model through a few loops of iteration. The model is applied for the case of building on surface of half-space soil medium and satisfactory results are achieved in comparison to the results of more rigorous models and also code provisions. The model is also easily applicable for more complicated cases such as buildings on layered half-space soil media.

Key Words: Soil-Structure Interaction , Simplified Model , Complex Eigenvalue Analysis , Damping Ratio , Natural Frequency , Cone Models

Introduction:

It is well-known that the dynamic properties of structures are influenced by flexibility of soil under them due to Soil-Structure Interaction (SSI). As a result, the soil-structure system usually has a longer natural period and higher damping ratio than the structure would have in the fixed-base state. Especially, the latter effect is considerable due the radiation damping in the soil and may significantly affect the structural response to dynamic loads. The importance of SSI effect as well as the complexity of the phenomenon have made it the subject of several researches for the last three decades.[1-19] It also has found its way into some seismic codes and provisions as simplified guidelines.[20,21] Among different treatments with the subject, the possible application of modal analysis to interacting soil-structure system, with its inherent advantages, has attracted the attention of a number of researchers.[4-11] As a key parameter in modal analysis, many researchers have put effort against estimating the modal damping coefficients. This has been mainly done by matching the rigorous and normal mode solutions of transfer function[8] or by using energy methods[5-7] which are based on the assumption of equal undamped and damped mode shapes. However, application of complex eigenvalue analysis is rare[4,12,15] and almost limited to the case of structure on surface of soil half-space with frequency independent impedance functions. The complex eigenvalue analysis method was used by the authors[16] for the case of structure located on surface of homogeneous half-space replaced by frequency dependent springs and dashpots using Cone Models.[22] The method was then extended to the case of layer on flexible half-space.[17] Here, as a further step, a simplified formulation based on a 3 degrees of freedom (DOF) replacement model is introduced which can predict the modal damping

* Graduate Student — Dep. of Architecture, Graduate School of Eng., Nagoya Univ, M.Sc.
** Professor — Center for Cooperative Research in Advanced Science & Tech., Nagoya Univ., Dr. Eng.
*** Research Associate — Dep. of Architecture, Graduate School of Eng., Nagoya Univ., Dr. Eng.

ratios of soil-structure systems with sufficient accuracy. There are also some other formulations based on well-established simplified models in the literature[2,3] which are indeed the basis of current seismic code provisions. However, the present formulation is based on the complex eigenvalue analysis concept by using frequency dependent dynamic stiffness for soil. Moreover, it considers the effect of foundation's mass and also allows the hysteretic form of damping to be used as the material damping in both soil and structure which is a more realistic model than the commonly used viscous form of damping.[23,24] The proposed model is able to capture the results of more rigorous model already used by the authors[15-17] with much less effort. Moreover, it may be used for more general soil-structure systems.

The Basic Soil-Structure Model:

Figure 1 shows the conventional soil-structure model which has been used by several researchers. The structure is modeled as a shear building and the soil is replaced by sway and rocking springs and dashpots. It is common practice among researchers to use frequency independent coefficients for the soil representative springs and dashpots because of its simplicity.[1,2,4,5,8] However, although the idea may practically work for the case of building on surface of homogeneous soil half-space, generally it is not applicable for the layered sites where the dynamic stiffness of soil varies drastically with frequency.[19]

Generally, the standard eigenvalue analysis is not applicable to the soil-structure system due to the

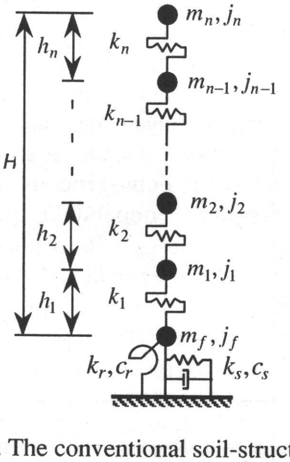


Figure 1. The conventional soil-structure model

difference in the nature of damping mechanism in the bounded structure and unbounded soil.[7-9] It is because the damping matrix wouldn't be in a form to allow the system posses classical modes of vibration.[25,26] Although there are well-established techniques for handling even non-classical damped systems,[10,27] they are not applicable when the frequency dependency of soil stiffness is also considered. Also, the application of such techniques is limited to cases with real stiffness matrices where the use of complex damping for modeling the material damping in the soil or structure is not applicable. On the other hand, the results of experimental studies on structures are more compatible with the concept of the hysteretic damping model (complex damping) than the commonly used viscous form of damping.[23] Also, the hysteretic damping model has been proposed as the best possibility for modeling the material damping in the soil.[24] As an alternative method capable to deal with all above mentioned problems, explicit presentation of determinant of the stiffness matrix was used by the authors.[15-17] The case of building with the same mass and stiffness for all stories was studied and the determinant of the stiffness matrix of the system was expressed by polynomials explicitly through the cofactor expansion method. However, special techniques are required to solve the equations in the complex plane. Additionally, the method is limited to the case of buildings with uniform distribution of mass and stiffness in height.

The Replacement 3DOF Model and Formulations:

The model of Fig.1 discussed in the previous section is replaced by a much simpler 3DOF model in this section. Figure 2 shows the simplified model where the superstructure -building- is replaced by its modal effective mass, $m_{str.}$, and modal effective stiffness, $k_{str.}$, providing the same modal frequency as the original multi degree of freedom (MDOF) model, as shown in Fig.1, in the fixed base state.

$$\omega_{fix} = \sqrt{k_{str.}/m_{str.}} \tag{1}$$

The effective mass for the qth mode is defined as

$$m_{str.} = \frac{\left[\sum_{p=1}^{n} m_p\, \varphi_{pq}\right]^2}{\sum_{p=1}^{n} m_p\, \varphi_{pq}^{\,2}} \tag{2}$$

where m_p is the mass of the pth story and φ_{pq} is the

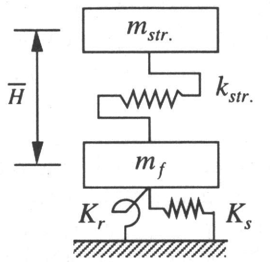


Figure 2. The simplified model

amplitude at pth story in the qth mode of vibration computed for the fixed-base MDOF model. Also, the qth mode's effective height, $\overline{H}$, is defined as

$$\overline{H}=\frac{\sum_{p=1}^{n}\left[m_p\,\varphi_{pq}\left(\sum_{i=1}^{p}h_i\right)\right]}{\sum_{p=1}^{n}m_p\,\varphi_{pq}} \qquad (3)$$

The ratio of the modal effective mass and height factors,as defined in (2) and (3), to the total mass and height of the structure

$$m_{tot.}=\sum_{i=1}^{n}m_i \qquad (4a)$$

$$H=\sum_{i=1}^{n}h_i \qquad (4b)$$

are referred to as the effective mass and height ratios in this text and are shown by λ_m and λ_H, respectively.

$$\lambda_m=\frac{m_{str.}}{m_{tot.}}\ ,\ \lambda_H=\frac{\overline{H}}{H} \qquad (5)$$

The foundation is represented by mass m_f in the simplified model and the springs K_s and K_r are considered as the frequency dependent dynamic stiffness of soil in the sway and rocking DOF, respectively. It should be emphasized that each of K_s and K_r has been replaced for both of the related spring and dashpot in the model of Fig.1 and consequently has complex value stiffness as follows:

$$K_s=k_s+i\omega\,c_s \qquad (6a)$$

$$K_r=k_r+i\omega\,c_r \qquad (6b)$$

where ω is the circular frequency of vibration and $i=\sqrt{-1}$.

Introducing the dimensionless parameters

$$\alpha=\frac{m_f}{m_{str.}}\ ,\ \beta=\frac{k_{str.}}{K_s}\ ,\ \gamma=\frac{K_r}{k_{str.}\cdot\overline{H}^2} \qquad (7)$$

the mass and stiffness matrices of the simplified model can be written as follows.

$$M=m_{str.}\begin{bmatrix}1 & 0 & \overline{H}\\ 0 & \alpha & 0\\ \overline{H} & 0 & \overline{H}^2\end{bmatrix} \qquad (8a)$$

$$K=K_s\begin{bmatrix}\beta & -\beta & 0\\ -\beta & 1+\beta & 0\\ 0 & 0 & \beta\gamma\overline{H}^2\end{bmatrix} \qquad (8b)$$

The first complex eigenfrequency of the system then will be

$$\omega=\omega_{fix}\sqrt{\frac{2\gamma}{\zeta+\sqrt{\zeta^2-4\alpha\beta\gamma(1+\gamma)}}} \qquad (9a)$$

where

$$\zeta=1+\gamma+\beta\gamma+\alpha\beta\gamma \qquad (9b)$$

Using Cone Models, the coefficients of the soil springs and dashpots may be expressed as follows

$$k_s=K_H\cdot k_H\ ,\ c_s=K_H\cdot c_H \qquad (10a)$$

$$k_r=K_\theta\cdot k_\theta\ ,\ c_r=K_\theta\cdot c_\theta \qquad (10b)$$

where k_H, k_θ, c_H and c_θ are the frequency dependent dynamic coefficients and K_H and K_θ are the static stiffness of disk located on surface of a homogeneous half-space for the sway and rocking DOF, respectively, as follows

$$K_H=\frac{8\rho V_s^{\,2}r}{2-\nu} \qquad (11a)$$

$$K_\theta=\frac{8\rho V_s^{\,2}r^3}{3(1-\nu)} \qquad (11b)$$

in which ρ, V_s and ν are respectively the specific mass, shear wave velocity and Poisson's ratio of soil and r is the radius of the circular foundation. The equivalent radius may be used in the case of rectangular foundations by matching the area or moment of inertia of the foundation with a circular foundation replacement for the sway or rocking DOF, respectively.[22] Using (1), (6) and (10), parameters β and γ in (7) are written in the following form

$$\beta=m_{str.}\omega_{fix}^{\ 2}/K_H(k_H+i\omega\,c_H) \qquad (12a)$$

$$\gamma=K_\theta(k_\theta+i\omega\,c_\theta)/\overline{H}^2\,m_{str.}\omega_{fix}^{\ 2} \qquad (12b)$$

Also, by introducing two other dimensionless parameters as

$$\overline{m}=m_{tot.}/\rho r^2 H \qquad (13a)$$

$$(a_0)_{fix}=r\omega_{fix}/V_s \qquad (13b)$$

finally, (12) may be rewritten as follows

$$\beta = \left[\frac{2-\nu}{8}\overline{m}\,\lambda_m \frac{H}{r}(a_0)^2_{fix}\right](k_H + i\omega c_H)^{-1} \quad (14a)$$

$$\gamma = \left[\frac{3(1-\nu)}{8}\overline{m}\,\lambda_m \lambda_H^2\left(\frac{H}{r}\right)^3 (a_0)^2_{fix}\right]^{-1}(k_\theta + i\omega c_\theta) \quad (14b)$$

The eigenvalue of the system can then be evaluated by using (9) and (14) by iteration while the dynamic stiffness coefficients of k_H, c_H, k_θ and c_θ are updated in each loop. Then, the damped frequency and damping ratio of the system will be evaluated as follows

$$\omega_d = \mathrm{Real}(\omega) \quad (15a)$$

$$\xi = \frac{\mathrm{Imag}(\omega)}{\mathrm{Abs}(\omega)} \quad (15b)$$

where Real(), Imag() and Abs() mean the real, imaginary and absolute values, respectively. The method is efficient and generally just a few iterations are required for the convergence into the result. At the end, it should be added that the material damping in the soil and structure may be included in the formulations as the hysteretic form of damping by using the correspondence principle,[22,23] i.e., just by replacing the stiffness of the structure and soil by

$$\hat{k}_{str.} = k_{str.}(1 + 2\xi_{str.}\, i) \quad (16a)$$

$$\hat{K}_s = K_s(1 + 2\xi_{soil}\, i) \quad (16b)$$

$$\hat{K}_r = K_r(1 + 2\xi_{soil}\, i) \quad (16c)$$

where $\xi_{str.}$ and ξ_{soil} are the material damping ratios in the structure and soil, respectively.

Application of the Model to the First Mode of Vibration:

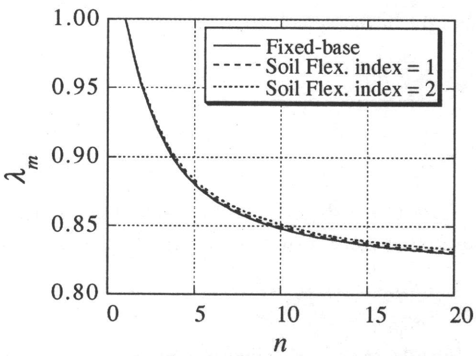
Generally, the first mode of vibration is the most important mode in the modal analysis of ordinary buildings. Also, the effect of SSI on the response of buildings may be taken into account with sufficient accuracy by considering only the change in the first mode's frequency and damping ratio due to SSI.[20,21] Moreover, it was also shown by the authors[16,28] that generally the higher modes of the soil-structure system have lower damping ratios due to lower interaction effect. The exception for this finding are the cases of flexible structures located on relatively stiff soil where SSI doesn't play any important role. Thus, the effect of SSI on the characteristics of the first mode of vibration of the soil-structure system is studied here using the introduced simplified model.

The concept of the model and formulations are also applicable for the higher modes of vibration by some minor modifications. However, they are not discussed herein.

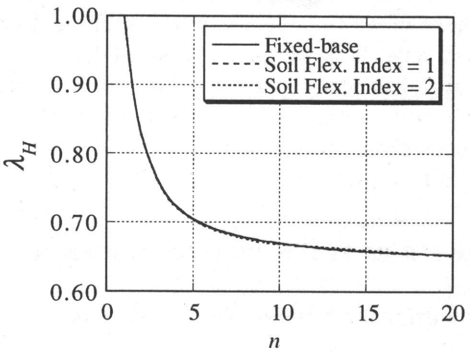
Before discussing the results, a point regarding buildings with low aspect ratios is noteworthy. Since the stories' mass moment of inertia have been neglected in the simplified model, it may cause an error in the results for such squat buildings in which the sum of the mass moment of inertia of stories is comparable or higher than the value of $m_{str.}\overline{H}^2$. This problem will be resolved with sufficient accuracy by replacing λ_H with $\hat{\lambda}_H$ as defined below.

$$\hat{\lambda}_H = \lambda_H \sqrt{1 + \frac{1}{4\lambda_H^2 (H/r)^2}} \quad (17)$$

This modification factor approaches to unity for higher aspect ratios very rapidly and doesn't introduce any specific error for the case of tall buildings.



(a) The effective mass ratio



(b) The effective height ratio

Figure 3. The effective mass and height ratios for different structure-soil stiffness ratios. (Soil Flexibility Index = $(a_0)_{fix} \cdot H/r$)

The non-dimensional parameters used in the formulation of the simplified model may be summarized as

$$(a_0)_{fix} \; , \; H/r \; , \; \overline{m} \; , \; \alpha \; , \; \lambda_m \; , \; \lambda_H \qquad (18)$$

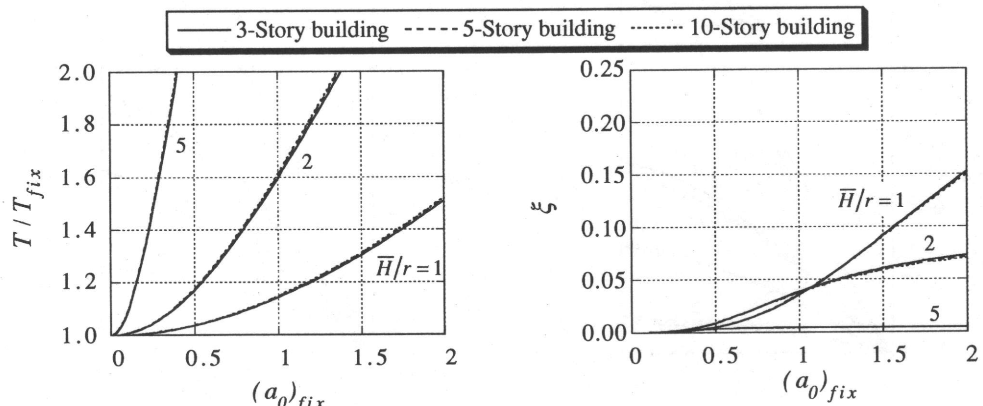
in addition to four soil dynamic stiffness coefficients which are frequency dependent and updated in each loop of iteration using Cone Modes. Among the parameters in (18), the first two parameters have been selected as the key parameters here. The third one may be set to a typical value of $\overline{m} = 0.5$ for ordinary buildings. The other three parameters can easily be evaluated for any specific case when the mass, stiffness and height of each story as well as the mass of the foundation are known. As an example, a building with the same mass, stiffness and height for all stories attached to a foundation whose mass is the same as stories is examined here. This is the same model as the one used by the authors in their previous studies.[15-17] Therefore, the results may be compared to those of the studies for evaluation of the proposed model. The variation of the last two parameters in (18) with the number of stories for such a case are shown in Fig.3. Since the existence of the soil affects the mode shapes of the structure and consequently the effective mass and height ratios, their variation should be taken into account in order to secure the reliability of the results. In this regard, the results for two levels of structure-soil stiffness ratio are also presented in Fig.3 in comparison with the results of the fixed-base model. As seen, the effect of soil flexibility on the mass and height effective ratios of the structure are negligible and the values related to the fixed-base model may be used with sufficient accuracy. The remaining parameter in (18), α, will also be evaluated as follows for such an example with the same mass for all stories and the foundation.

$$\alpha = 1/(n\,\lambda_m) \qquad (19)$$

Equation (19) and Fig. 3 reveal that the last three parameters in (18) are functions of the number of stories in the MDOF model. This may weaken the general applicability of the model. It is, however, shown here that despite the dependency of these parameters on the number of stories, the final results of the simplified model will be independent of this factor. It means that different sets of parameters α, λ_m and λ_H related to structural models with different number of stories lead to the same results for the change in the natural period and for the damping ratio of the system. However, for this purpose, instead of the aspect ratio of the building, the effective aspect ratio defined as

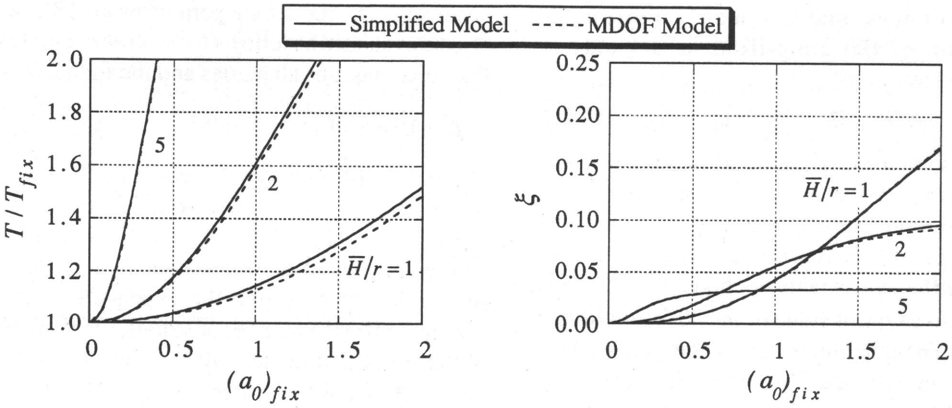
$$\frac{\overline{H}}{r} = \frac{H}{r} \cdot \lambda_H \qquad (20)$$

should be used as the key parameter along with $(a_0)_{fix}$. Figure 4 shows the results for three different sets of parameters α, λ_m and λ_H corresponding to buildings with three different number of stories. The results are shown for a wide range of $(a_0)_{fix}$ covering the systems with no SSI, $(a_0)_{fix}=0$, to the systems with severe SSI effect with $(a_0)_{fix}=2.0$.[28] It should be noted that material damping is not considered in either the soil or the structure for the drawing shown in Fig.4. As seen in Fig.4, the results are essentially independent of the number of stories. This allows to set the last three parameters in (18) to some typical



(a) The change in the natural period (b) The soil induced damping ratio

Figure 4. Comparison of the results for buildings with different number of stories
(No material damping is addressed in the soil and structure)



(a) The change in the natural period (b) The soil induced damping ratio

Figure 5. Comparison of the results of the simplified and MDOF models ($\xi_{soil} = 0.03$)

constant values. Here the following values are used

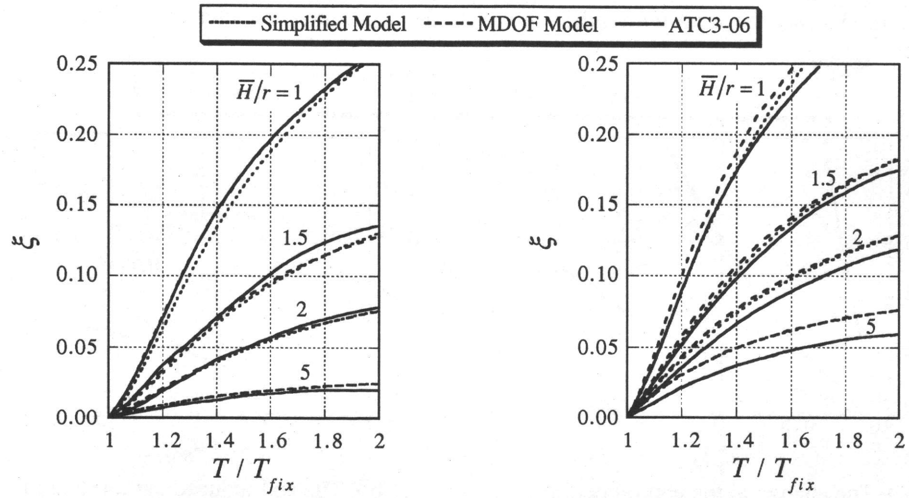
$$\alpha = 0.118 \ , \ \lambda_m = 0.848 \ , \ \lambda_H = 0.669 \qquad (21)$$

which belong to a 10-story building model.

Representative Results:

Using the values in (21), the results in the same manner as Fig.4 are drawn in Fig.5 in comparison with those for the MDOF model.[28] However, here a hysteretic material damping ratio of 3% has been considered in the soil. As shown, the results of the two models are almost identical, reinforcing the applicability of the simplified model. This model provides a more accurate approximation for the natural period of the system than those suggested by ATC3-06[20] and NEHRP[21]. This is mainly because of disregarding the effect of the floors' mass moment of inertia in the model used by ATC3-06 and NEHRP which may lead to significant errors for short and squat buildings.[28] Since the regulations of ATC3-06 and NEHRP are almost the same, they just will be referred to as ATC3-06 hereafter in this text.

The results of Fig.5 may be presented in a new format as the variation of the system's damping ratio with the change in the natural period of the system by omitting $(a_0)_{fix}$ between parts (a) and (b) of the figure. This format is generally more desirable from the practical structural design point of view. Figure 6 shows the results of this study in this new format



(a) $\xi_{soil} = 0.03$ (b) $\xi_{soil} = 0.10$

Figure 6. Comparison of the simplified model results with the MDOF and seismic code provisions

for two levels of material damping in the soil. Also drawn in the figure are the results of the MDOF model[28] and the graphs suggested by ATC3-06. It should be mentioned that the values of 3 and 10 percent as the material damping in the soil have been selected consistent with two soil strain levels introduced by ATC3-06. Empirical relationships have been used in this regard.[29,30] The results of the simplified model in Fig.6 show full agreement with those of the MDOF model which are in turn in good agreement with ATC3-06 results. In fact, the simplified model and the MDOF model practically lead to the same results and for some cases, including tall buildings, the results of two models are even not distinguishable from each other in Fig.6. Also, the gap between the results of these two models with those of ATC3-06 can well be explained by the empirical nature of the used relationships between the expected material damping and the strain level in the soil. This confirms the reliability of the proposed model and its applicability. The model also has the capability to be used for more complicated cases such as buildings located on layered half-space soil media,[28] for which no detailed information is currently available.

For completing the discussion, the effect of soil-structure interaction on the structural damping itself should also be addressed. As the soil flexibility changes the vibration modes of the structure, the amount of energy dissipated in the structure itself also changes.[14] This leads to lower internal (material) damping in the structure for the soil-structure system due to reduction of curvature. Although in lightly damped structures this less desirable effect would be negligible, for the case of structures with heavy internal damping, the effect may be considerable.[12] Also, it has been pointed out that in the cases of structures with large internal damping, the loss of damping due to this effect may be greater than the gain due to radiation damping in the soil.[4,12] This phenomenon is more probable with slender structures.[3] However, it has been shown that this possibility doesn't occur when structural damping is assumed to be hysteretic rather than viscous.[18] The variation of structural damping due to soil flexibility is also studied here. The structural damping reduction factor, δ, is defined as follows in this regard.

$$\delta = \frac{\bar{\xi}_{str.}}{\xi_{str.}} \tag{22}$$

where $\xi_{str.}$ and $\bar{\xi}_{str.}$ are the internal damping ratio of the structure in the fixed-base state and when located on flexible soil, respectively. The variation of δ has been approximated by some researchers as follows[13,29]

$$\delta = \left(\frac{T_{fix}}{T}\right)^3 \tag{23}$$

which is valid for systems with viscous type of material damping in the structure. The same approximation has been also used by ATC3-06. The assumption of hysteretic type of damping for the structure, as adopted here, however, leads to an exponent 2 (instead of 3) in the right-hand side of (23)[19], i.e.,

$$\delta = \left(\frac{T_{fix}}{T}\right)^2 \tag{24}$$

Figure 7 shows the results of this study for systems

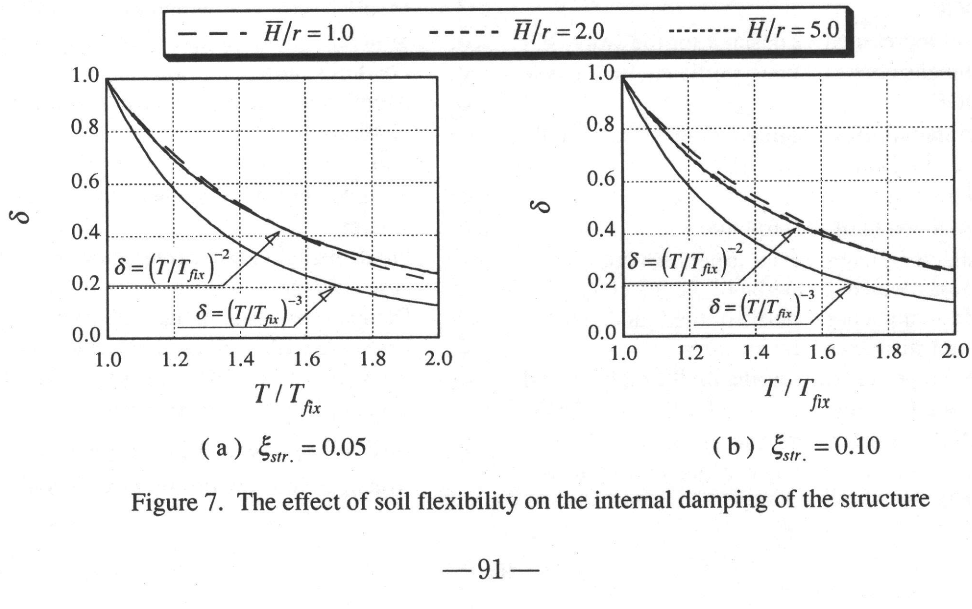


Figure 7. The effect of soil flexibility on the internal damping of the structure

with two different levels of internal damping and with different values of effective aspect ratios for the structure. Also drawn in this figure are the curves of Eqs. (23) and (24). As shown, the results of this study are well matched with Eq.(24) as expected for a system with hysteretic material damping model. Especially for slender buildings the results almost coincide the curve of Eq.(24) and they are not distinguishable from each others.

Conclusion:

A simple 3DOF model was introduced as the replacement for more complicated soil-structure systems. The dynamic properties of the model for the first mode of vibration were calculated through the parametric complex eigenvalue analysis. Code type graphs for the relationship between the soil induced damping ratios and the change in the period of building were drawn which are in good agreement with the regulations of ATC3-06 and NEHRP. Although the model doesn't consider for the number of stories, the results are in full agreement with those of the MDOF building-soil models with different number of stories but the same effective aspect ratio of building. The model is also applicable to more sophisticated cases such as building on a layered half-space soil medium, for which no detailed information is currently available.

Nomenclature:

$(a_0)_{fix}$: Dimensionless frequency computed for the fixed base state of the structure
c_r : Soil representative dashpot for the rocking DOF
c_s : Soil representative dashpot for the sway DOF
c_H : Dimensionless damper coefficient for the sway DOF
c_θ : Dimensionless damper coefficient for the rocking DOF
h_i : Height of the ith story
H : Total height of the structure
$\overline{H}$: Effective height of the structure, (Eq.3)
j_p : Mass moment of inertia of the pth story
j_f : Mass moment of inertia of the foundation
k_i : Total stiffness of the ith story
k_r : Soil representative spring for the rocking DOF
k_s : Soil representative spring for the sway DOF
$k_{str.}$: Modal effective stiffness of the structure
k_H : Dimensionless spring coefficient for the sway DOF
k_θ : Dimensionless spring coefficient for the rocking DOF
K : Stiffness matrix
K_H : Static stiffness of disk on surface of soil half-space in the sway DOF
K_θ : Static stiffness of disk on surface of soil half-space in the rocking DOF
K_s : Generalized dynamic stiffness of the soil in the sway DOF (Eq.6a)
K_r : Generalized dynamic stiffness of the soil in the rocking DOF (Eq.6b)
$\hat{k}_{str.}$: Complex modal stiffness of the structure (Eq.16a)
$\hat{K}_s$: Complex dynamic stiffness of the soil in the sway DOF (Eq.16b)
$\hat{K}_r$: Complex dynamic stiffness of the soil in the rocking DOF (Eq.16c)
m_f : Mass of the foundation
m_p : Mass of the pth story
$m_{str.}$: Modal effective mass of the structure (Eq.2)
$m_{tot.}$: Total mass of the structure
M : Mass matrix
$\overline{m}$: Structure-soil mass ratio index (Eq.13a)
n : Number of stories
r : (Equivalent) Radius of circular foundation
V_s : Shear wave velocity in soil

α : Foundation-structure mass ratio (Eq.7)
β : Structure-soil lateral stiffness ratio (Eq.7)
γ : Structure-soil rotational stiffness ratio (Eq.7)
ζ : Dimensionless parameter as defined in Eq.9b
λ_m : Effective mass ratio of the structure (Eq.5)
λ_H : Effective height ratio of the structure (Eq.5)
$\hat{\lambda}_H$: Modified effective height ratio (Eq.17)
δ : Structural damping reduction factor (Eq.22)
ν : Poisson's ratio in the soil
φ_{pq} : Amplitude at the pth story in the qth mode of vibration
ρ : Soil mass density
ω : Complex eigenfrequency of the soil-structure system
ω_{fix} : Fundamental circular frequency of the fixed-base structure
ω_d : Damped frequency of the soil-structure system
ξ : Damping ratio of the soil-structure system
$\xi_{str.}$: Internal (material) damping ratio in the structure at the fixed-base state
$\overline{\xi}_{str.}$: Internal (material) damping ratio in the structure when located on flexible soil

References:

1) Parmelee, R.A., Perelman, D.S. and Lee, S.L., "Seismic response of multiple-story structures on flexible foundations", *Bull. Seism. Soc. Amer.*, 1969, Vol.59, No.3,1061-1070

2) Sarrazin, M.A., Roesset, J.M. and Whitman, R.V., "Dynamic soil-structure interaction", *Struct. Engng. Div., ASCE*, 1972, Vol.98, ST7, 1525-1544

3) Veletsos, A.S. and Meek, J.W., "Dynamic behaviour of building-foundation systems", *Earthq. Engng. Struct. Dynam.*, 1974, Vol.3, 121-138

4) Jennings, P.C. & Bielak, J., "Dynamics of building-soil interaction", *Bull. Seism. Soc. Amer.*, 1973, Vol.63, 9-48

5) Bielak, J., "Modal analysis for building-soil interaction", *Engng. Mech. Div., ASCE*, 1976, Vol.102, EM5, 771-786

6) Novak, M., "Effect of soil on structural response to wind and earthquake", *Earthq. Engng. Struct. Dynam.*, 1974, Vol.3, 79-96

7) Roesset, J.M., Whitman, R.V. and Dobry, R., "Modal analysis for structures with foundation interaction", *Struct. Engng. Div., ASCE*, 1973, Vol.99, 399-415

8) Tsai, N.C., "Modal damping for soil-structure interaction", *Engng. Mech. Div., ASCE*, 1974, Vol.100, 323-341

9) Triall-Nash, R.W., "Modal damping in the dynamic of systems with non-classical damping", *Earthq. Engng. Struct. Dynam.*, 1981, Vol.9, 153-169

10) Veletsos, A.S. and Ventura, C., "Modal analysis of non-classically damped linear systems", *Earthq. Engng. Struct. Dynam.*, 1986, Vol.14, 217-243

11) Chopra, A.K. and Gutierrez, J.A., "Earthquake response analysis of multistorey building including foundation interaction", *Earthq. Engng. Struct. Dynam.*, 1974, Vol.3,65-77

12) Novak, M. and El Hifnawy, L., "Effect of soil-structure interaction on damping of structures", *Earthq. Engng. Struct. Dynam.*, 1983, Vol.11, 595-621

13) Veletsos, A.S. and Meek, J.W., *Dynamic behavior of building-foundation systems*, Report No.20, Dept. of Civil Engng., Rice Univ., Houston, Texas, 1973

14) Novak, M., "Additional note on the effect of soil on structural response", *Earthq. Engng. Struct. Dynam.*, 1975, Vol.3, 312-315

15) Fukuwa, N., Ghannad, M.A. & Yagi, S., "A study on the effect of soil-structure interaction on the eigenproperty of structure", *J. Struct. Constr. Engng., Transactions of Arch. Inst. of Japan (AIJ)*, 1995, No.475, 35-44, (In Japanese)

16) Fukuwa, N., Ghannad, M.A., "Soil-structure interaction effect on the eigenproperties of structure", *11WCEE, Acapulco, Mexico*, 1996, paper no. 949

17) Ghannad, M.A. and Fukuwa, N., "The effect of soil profile on the dynamic properties of soil-structure systems", *Annual meeting of Arch. Inst. of Japan (AIJ)*, 1997, Vol.B-2, 429-430

18) Aviles, J. and Perez-Rocha, L.E., "Evaluation of interaction effects on the system period and the system damping due to foundation embedment and layer depth", *Soil Dynamics and Earthq. Engng.*, 1996, Vol.15, 11-25

19) Wolf, J.P. *Dynamic Soil-Structure Interaction*, Prentice-Hall, Englewood Cliffs, New Jersey, 1985

20) Applied Technology Council (ATC), *Tentative provisions for the development of seismic regulations for buildings, ATC Publications, ATC3-06*, 1978

21) The Building Seismic Safety Council for the Federal Emergency Management Agency, *NEHRP Recommended provisions for the developement of seismic regulations for new buildings*, 1994

22) Wolf, J.P. *Foundation Vibration Analysis Using Simplified Physical Models*, Prentice-Hall, Englewood Cliffs, New Jersey, 1994

23) Clough, R.W. & Penzien, J. *Dynamics of Structures*, McGraw-Hill, New York, 2/E, 1993

24) Kausel, E. & Roesset, J., "Dynamic stiffness of circular foundations", *Engng. Mech. Div., ASCE*, 1974, Vol.101, 771-785

25) Foss, K.A., "Co-ordinates which uncouple the equations of motion of damped linear dynamic systems", *Appl. Mech. Div., ASME*, 1958, Vol.25, 361-366

26) Caughy, T.K., "Classical normal modes in damped linear systems", *Appl. Mech. Div., ASME*, 1960, Vol.27, 269-271

27) Hurty, W.C. and Rubinstein, M.F., *Dynamics of Structures*, Prentice-Hall Inc., 1964

28) Ghannad, M.A., *A Study on the Effect of Soil-Structure Interaction on the Dynamic Properties of Structures Using Simplified Methods*, Ph.D. Thesis, Nagoya University, 1998

29) Architectural Institute of Japan (AIJ), *An Introduction to Dynamic Soil-Structure Interaction*, 1996 (In Japanese)

30) Kokusho, T., "Cyclic triaxial test of dynamic soil properties for wide strain range", *Soil and Foundations*, 1980, Vol.20, 45-60