

国土数値情報を活用した地震時地盤増幅度の推定

ESTIMATION OF SOIL AMPLIFICATION FACTOR USING
DIGITAL NATIONAL LAND INFORMATION

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In order to evaluate the seismic risk in the area where soil data are not well furnished, an equation to estimate soil amplification factor using Digital national land information is proposed. The soil amplification factor is evaluated from the soil response ratio between soil surface and bedrock using the results of earthquake damage estimation conducted by Aichi prefecture and Nagoya city. Using regression analysis, an equation with ground level, land-form, surface soil condition is derived in order to estimate soil amplification factor. The material nonlinearity of soil is taken into account by modifying the acceleration amplification factor using maximum input acceleration on bedrock.

Keywords: Seismic ground motion, Soil amplification factor, Digital national land information, Earthquake disaster prevention, Material nonlinearity
地震動、地盤増幅度、国土数値情報、地震防災、材料非線形性

1. はじめに

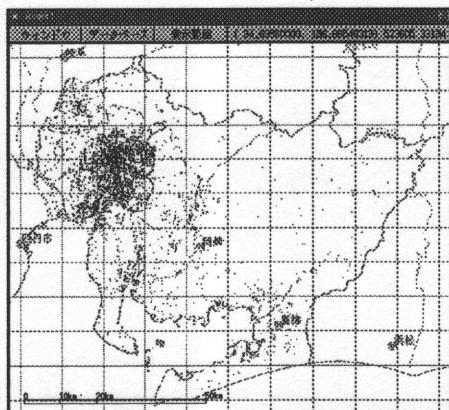
地震時の地盤の揺れの大きさを正確に予測することは、地震防災や建築物の耐震設計を行う上での基本である。しかし、地震時の地盤挙動の予測に必要となる地盤速度構造に関する資料は限られており、次善のデータであるボーリングデータも大都市域に集中している。一例として、図1に愛知県内で得られているボーリング調査位置を125mメッシュデータとして示すが¹⁾、図のように地盤資料は一部の地域に集中しており、県下約330,000メッシュに対して約13,000資料、すなわち約1/25のエリアをカバーしているにすぎない。ちなみに、名古屋市内では約20,000メッシュに対して約4,000資料が収集されているので²⁾、名古屋市外では名古屋市内に比べてワンオーダー低い密度でしか地盤資料が存在しないことになる。したがって、既存の地盤資料を活用して地盤の速度構造を推定し、地盤の応答解析を通して地盤増幅度を評価するという正攻法³⁾は、地盤資料の充実している大都市域を除くと適用しづらい。そこで本論では、全国的に整備されている国土数値情報⁴⁾に基づいた地盤増幅度の推定

式を提案する。

地盤増幅度の簡易推定法として実務的にしばしば用いられているのは、金井⁵⁾の地盤の最大増幅度 $G(T_G)$ と卓越周期 T_G との統計的関係、

$$G(T_G) = 5\sqrt{T_G} \quad (1)$$

や、翠川⁶⁾による地盤の速度増幅度 A_v (入射波最大値に

図1 愛知県内のボーリング調査位置¹⁾

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に対する地表面最大値の比)と地盤深さ30mまでの平均S波速度 V_s との関係式、

$$A_V = \text{Max} (2.5, 170 V_s^{-0.6}) \quad (2)$$

である。(2)式は応答計算に基づいて誘導されたものであるが、さらに翠川ら⁹⁾は1987年千葉県東方沖地震の記録の分析に基づいて類似の関係式を導いている。また、萩尾ら⁸⁾は解析的検討に基づいて工学基盤に対する表層地盤の増幅率を求め、地盤のS波速度構造を用いた推定式を与えている。しかし、これらの増幅度推定式を利用するには当該地盤の卓越周期やS波速度構造、平均S波速度などを知る必要があり、現状では十分なデータが備わっている地域は稀である点に問題が残される。地盤データの制約の問題を克服するため、松岡・翠川⁹⁾は関東地方の459地点でのS波速度資料に基づいて、平均S波速度と国土数値情報⁴⁾に含まれる地盤条件との関係を検討し、地表から深さ30mまでの地盤の平均S波速度を国土数値情報に基づいて推定する経験式を提案した。さらに、この経験式と前述の平均S波速度に基づく速度増幅度推定式⁷⁾を組み合わせることにより、関東地方の広域の震度分布予測を行っている¹⁰⁾。翠川らのこれらの一連の研究は、全国的に整備されている国土数値情報を広域の地震被害予測に有効活用しようとする先駆的な研究である。なお、国土数値情報は、地盤に関するデータとしては、標高・地形・表層地質・土壤などを有し、1kmメッシュデータ（標高のみ50, 250mメッシュ）として全国一律に整備されている。

本論では、愛知県¹⁾および名古屋市¹¹⁾での地震被害想定において得られた地震動予測結果を利用し、地表と地震基盤（S波速度3km/s相当の基盤面）の最大振幅の比を地盤増幅度と定義して、国土数値情報の標高・地形・表層地質の岩石区分を説明変数とした重回帰分析を行い、地盤増幅度の推定式を誘導する。地域の地震被害想定結果を有効利用している点、S波速度等を介すことなく直接地震基盤からの増幅度を求めている点、データの欠落箇所が多い表層地質の時代を敢えて指標とせず地形等欠落の少ない情報を重視している点などに特徴がある。また、本論では速度増幅度と加速度増幅度の2つの推定式を作成しているが、加速度増幅度には地盤の材料非線形性の影響も考慮している。

2. 地盤増幅度の定義

地表の最大応答値を地震基盤（S波速度3km/s相当の基盤面）における入射波最大値の2倍で除したものを地盤増幅度と定義し、速度と加速度について地盤増幅度を求める。地盤の地震動増幅には周期特性が重要な役割を果たすが、本論では国土数値情報というマクロな情報に基づいて簡易的に応答増幅効果を評価することを主目的として、最大値の比で増幅効果を表現することとする。

名古屋地域では、愛知県および名古屋市において地震

被害想定が実施されており、この中に各種想定地震に対する地震動レベルが記されている^{1), 11)}。両調査では同一の地震動評価方法が用いられており、地盤モデルも統一された基準で設定されている。愛知県は想定東海地震を、名古屋市はこれに加えて1891年濃尾地震と1944年東南海地震を想定地震としている。ここで用いられている地震動の評価方法は以下の通りである。

まず、各地震の断層モデルを定義し¹²⁾、翠川・小林の方法¹³⁾に則って地震基盤における減衰5%の速度応答スペクトル S_v を推定する。地震基盤での入射波最大値は、速度応答スペクトル S_v の積分値等（スペクトル強度あるいは修正スペクトル強度）と最大値との間の経験式（(3)式）を用いて算定する。

$$A_{max} = 0.3 \int_{0.1}^{2.5} S_A(T) dT \quad V_{max} = 0.3 \int_{0.1}^{2.5} S_V(T) dT \quad (3)$$

地盤の増幅特性については、工学基盤（S波速度700m/s相当の基盤面）以深は線形の重複反射解析により、工学基盤以浅は地盤の材料非線形性を考慮した等価線形重複反射解析により推定し、両伝達関数の積から地震基盤に対する地表の伝達関数を推定している。深い地盤構造は重力探査結果や爆破探査結果に基づいて、等S波速度の地層基底面等高線を推定している。一方、浅い地盤構造は既存のボーリング調査資料に基づいて計316種類の代表地盤モデル（名古屋市内は77代表地盤モデル）に区分・設定し、県および市内を500mメッシュ分割した上で、各メッシュ毎に代表地盤モデルを割り当てている。なお、工学基盤以浅の増幅特性評価には、代表的な数波の地震動を工学基盤位置に入射させた等価線形解析を実施して、幾つかの加速度レベルに対する平均伝達関数を予め用意しておき、工学基盤の加速度レベルに応じてこの伝達関数を補間して用いている。地表応答を評価するに当たっては、まず地震基盤における減衰5%の速度応答スペクトルを減衰補正した上で、この非減衰速度応答スペクトルを地震基盤の加速度フーリエ振幅スペクトルと見なし、前述の地震基盤に対する地表の伝達関数を乗じて、地表の加速度フーリエ振幅スペクトルを算定している。さらに、非減衰速度応答スペクトルを介して地表における減衰5%の速度応答スペクトルを評価し、地震基盤での場合と同様に(3)式を用いて速度応答スペクトルの積分値等から加速度・速度最大応答値を推定している。

図2および図3に、愛知県地震被害想定結果に基づく想定東海地震に対する地表の最大応答値と、地盤増幅度（地震基盤における入射波最大値の2倍に対する地表最大応答値の比）を加速度と速度について示す。地表応答は、震源に近い愛知県東部のみならず、震源から離れた濃尾平野や旧矢作川河口域などの沖積平野部でも大きくなっている。この結果、地震基盤に対する地盤増幅度は、沖積平野を中心として大きな値を示しており、この

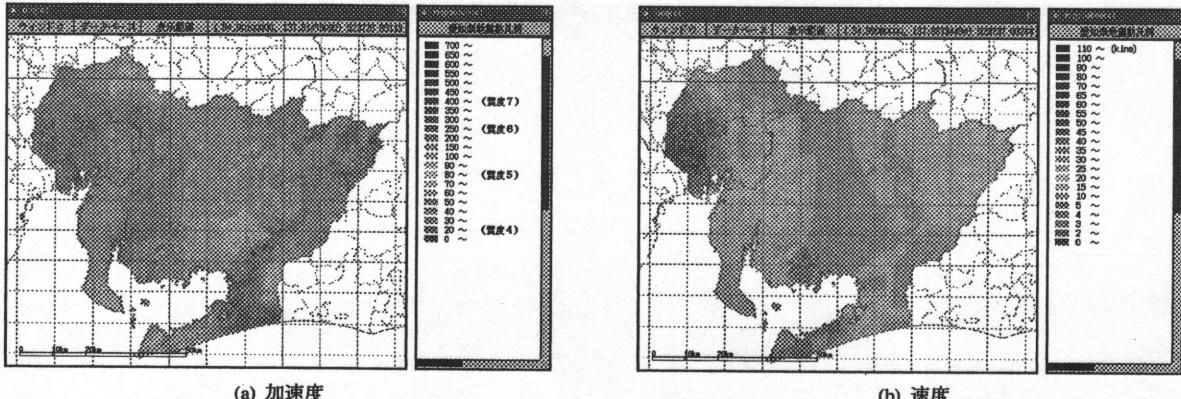


図2 想定東海地震に対する地表最大応答値（愛知県地震被害想定結果）

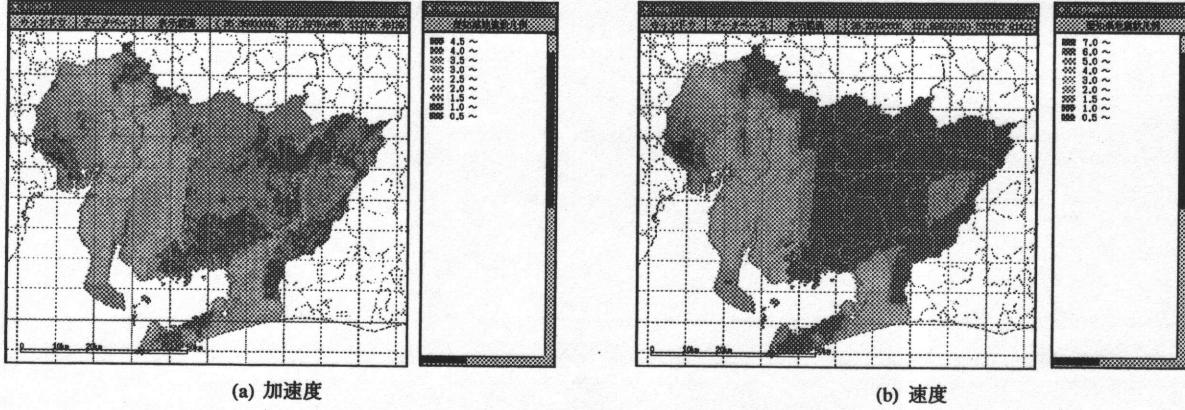


図3 愛知県地震被害想定結果に基づく想定東海地震に対する地盤増幅度

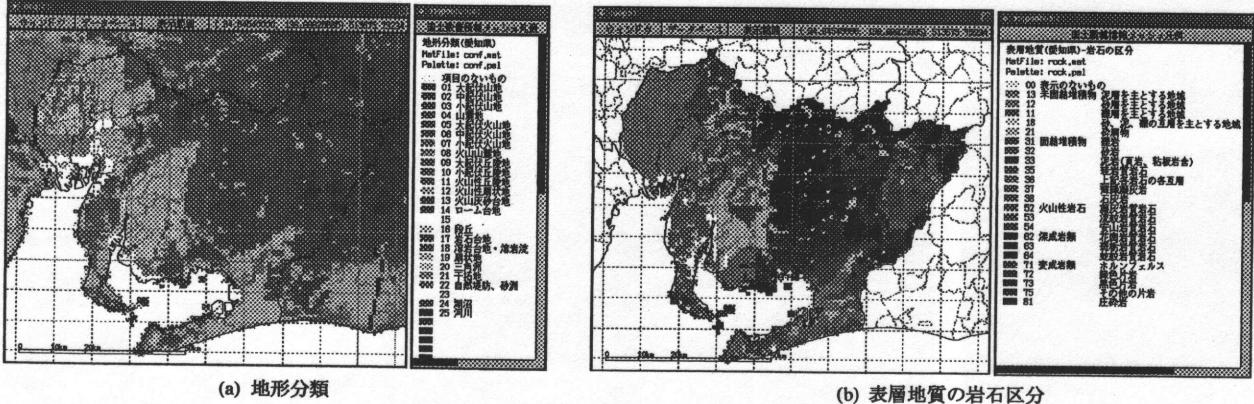
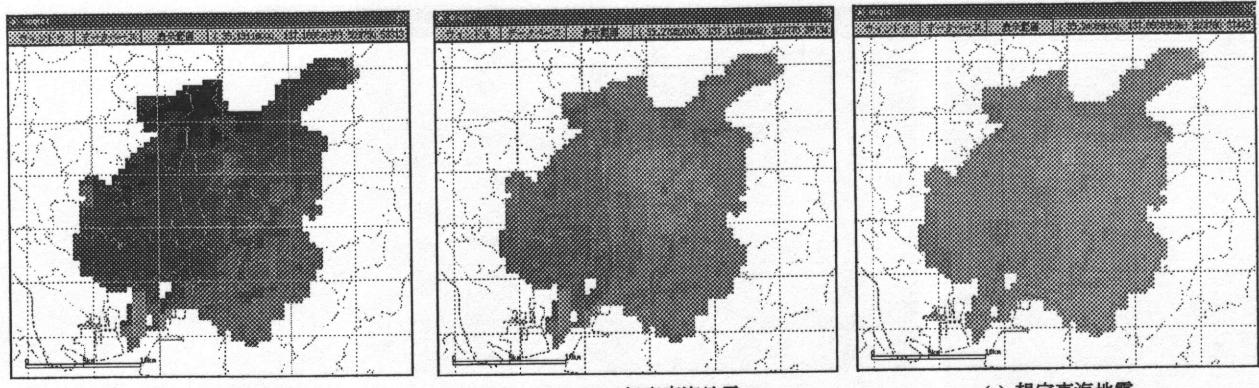


図4 国土数値情報による地盤情報

傾向は速度増幅度に顕著となっている。これは、速度成分では軟弱地盤での長周期域の増幅特性への寄与が大きいためである。図4に国土数値情報による愛知県内の地形分類と表層地質の岩石区分を示すが、図3の地盤増幅度と強い相関を示しており、国土数値情報に基づく地盤増幅度推定の可能性を示唆している。なお、想定東海地震に対する地震基盤での入射波の加速度レベルは愛知県内で20～70Gal程度である。

つぎに、図5～7に名古屋市地震被害想定結果に基づく1891年濃尾地震、1944年東南海地震、想定東海地震に対する地表の加速度最大応答値、地震基盤（S波速度3km/s相当の基盤面）および工学基盤（S波速度700m/s相当の基

盤面：第三紀層上面位置）に対する地表の加速度増幅度を示す。また、図8には地震基盤に対する速度の地盤増幅度を示す。ちなみに、地表加速度応答レベルは3地震で大きく異なり、濃尾地震では市北部で800Galを超え、東南海地震、想定東海地震では市西部でそれぞれ約350Gal、約200Galを示している。また、地震基盤に対する加速度増幅度は、名古屋市西部の沖積地盤において3地震での差異が大きい。この差異は、図7より工学基盤に対する加速度増幅度の違いによっており、地震動レベルの違いによる地盤の材料非線形性の影響と推察される。これに対して、地震基盤に対する速度増幅度は3地震で同様の傾向を示しており、地震動レベルによる差異は小さい。これ

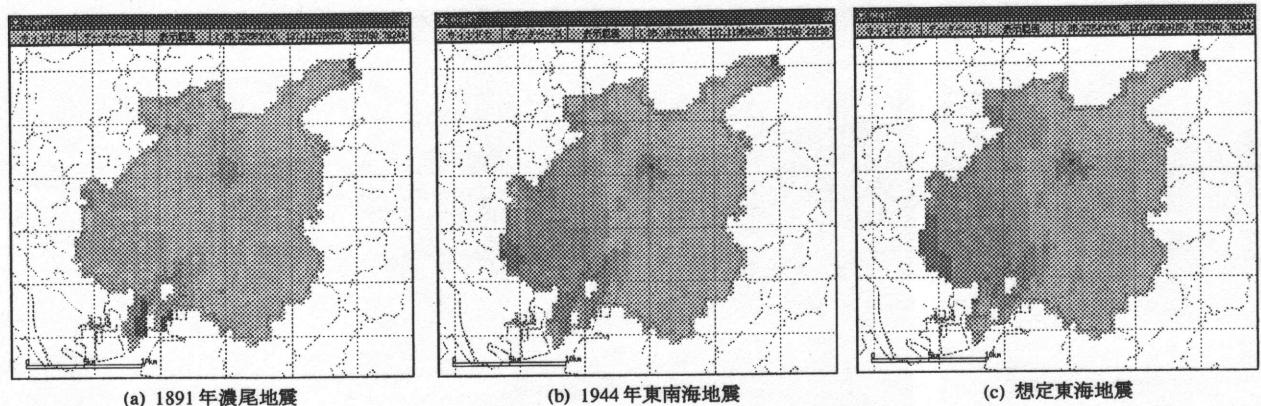


(a) 1891年濃尾地震

(b) 1944年東南海地震

(c) 想定東海地震

図5 地表の加速度最大応答値（名古屋市地震被害想定結果）

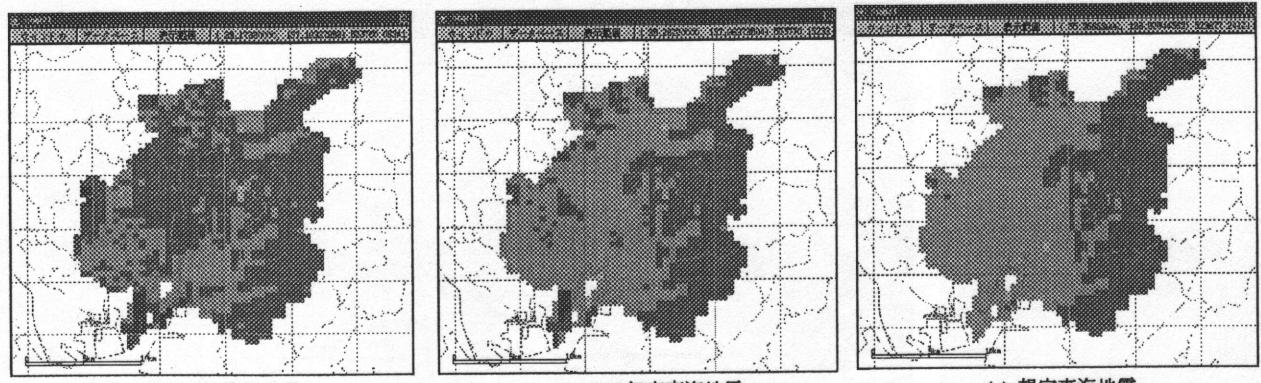


(a) 1891年濃尾地震

(b) 1944年東南海地震

(c) 想定東海地震

図6 名古屋市地震被害想定結果に基づく地震基盤に対する地盤増幅度（加速度）

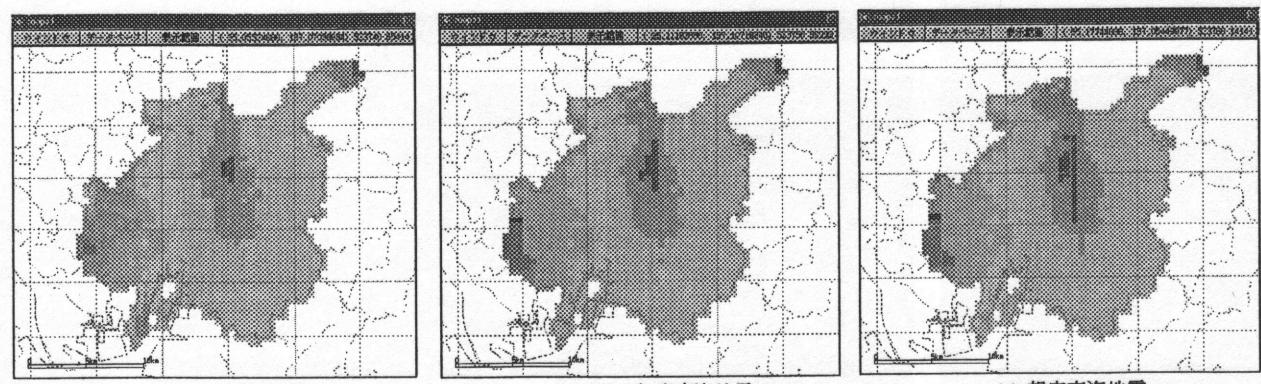


(a) 1891年濃尾地震

(b) 1944年東南海地震

(c) 想定東海地震

図7 名古屋市地震被害想定結果に基づく工学基盤に対する地盤増幅度（加速度）



(a) 1891年濃尾地震

(b) 1944年東南海地震

(c) 想定東海地震

図8 名古屋市地震被害想定結果に基づく地震基盤に対する地盤増幅度（速度）

は、地盤の塑性化に伴う長周期化と減衰増大が、加速度応答に対しては応答を低減させるが、速度応答に関しては低振動数域の寄与が大きいので両効果が相殺したためと考えられる。なお、地震基盤における入射波の加速度レベルは、3地震全体としては25~125Gal程度の間に分布している。

以上の結果から、愛知県の場合には、山地や沖積平野を含むため地形や表層地質等の種類が豊富であるが、想定地震が想定東海地震に限定されているので応答レベルの範囲が狭いという欠点があることが分かる。一方で、名古屋市の場合には地盤種類は少ないものの3つの想定地震での結果があるため、地震基盤位置の入射波加速度に幅がある。地震動の振幅レベル依存性が大きいのは沖積地盤の加速度増幅度であるので、名古屋市の結果の有効利用が期待できる。そこで、本論では以下に示す2段階で、国土数値情報に基づく地盤増幅度の推定を行う。

1) 愛知県の想定東海地震に対する地震動予測結果を用いて、国土数値情報の標高・地形・表層地質の岩石区分

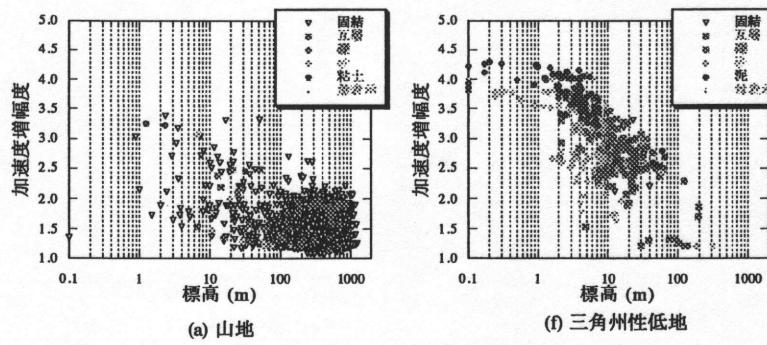


図9 標高と加速度増幅度の関係

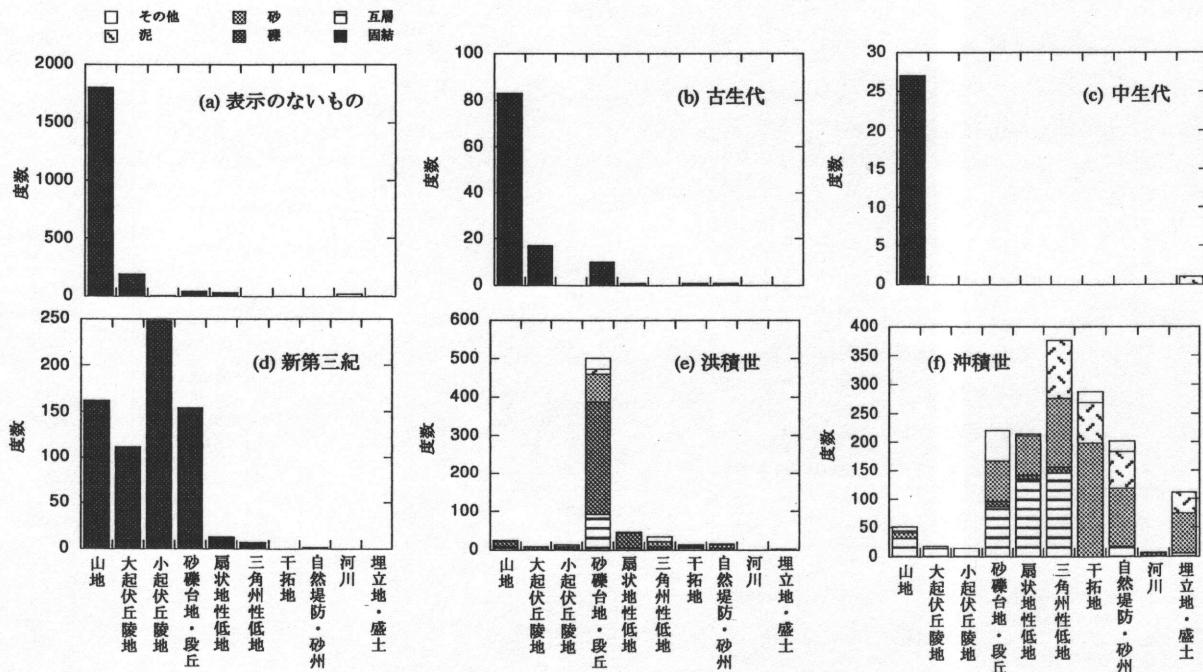


図10 表層地質の時代と地形

分を説明変数とした重回帰分析を行い、線形時の加速度・速度増幅度を推定する。

2) 名古屋市の3つの想定地震に対する予測結果を用いて、地震基盤での入射波加速度と加速度増幅度との関係を地形・表層地質の岩石区分毎に調べ、地盤材料の非線形化に伴う加速度増幅度の低減係数を求める。

3. 線形時の地盤増幅度の推定式

図3~4に示したように、地盤増幅度と国土数値情報の表層地質の岩石区分の間には強い相関が認められた。そこで、愛知県の想定東海地震に対する地震動予測結果を用いて地震基盤に対する加速度及び速度増幅度を算定し、国土数値情報に含まれる各種の説明変数に対して相関分析を行った結果、地盤増幅度は標高・地形・表層地質の岩石区分との相関が高いことが分かった。図9に、相関分析の一例として、山地および三角州低地における標高と加速度増幅度の関係を示す。図より、標高の常用対数と地盤増幅度との間には線形関係が認められる。標高

の対数を導入して地盤増幅度に対する沖積平野部での標高の重みを増すことにより、結果として、地盤増幅度に沖積層厚さを擬似的に反映させることができる。なお、国土数値情報の地形や表層地質の岩石区分はかなり細かく分類されているが、相関分析の結果幾つかの区分はまとめて取り扱っても支障のないことが分かったので、表1および表2の様に再分類した。図10に表層地質の時代別の地形ならびに岩石区分の分布を示す。図より、

表層地質の時代は他の指標（地形・表層地質の岩石区分）との相関が高いことが分かる。従って、特に愛知県においては欠落の多い表層地質の時代を敢えて指標として用いることなく、標高・地形・表層地質の岩石区分を説明変数とした地盤増幅度推定式を作成することにした。重回帰分析の結果は(4)式の通りである。

$$ARA_0 = 2.67 - 0.422 \log GL + \begin{bmatrix} 0.00 \\ 0.03 \\ 0.43 \\ 0.34 \\ 0.04 \\ 0.42 \\ 0.31 \\ 0.36 \\ 0.24 \\ 0.24 \end{bmatrix}_L + \begin{bmatrix} 0.00 \\ 0.22 \\ -0.04 \\ 0.24 \\ 0.67 \end{bmatrix}_S \quad (4)$$

$$ARV = 2.94 - 0.665 \log GL + \begin{bmatrix} 0.00 \\ -0.17 \\ 0.31 \\ -0.04 \\ -0.08 \\ 0.45 \\ 0.63 \\ 0.36 \\ -0.45 \\ -0.43 \end{bmatrix}_L + \begin{bmatrix} 0.00 \\ -0.08 \\ -0.13 \\ 0.33 \\ 1.41 \end{bmatrix}_S$$

ここに、 ARA_0 は線形時の加速度増幅度、 ARV は速度増幅度、 GL は標高（単位m）を表す。また、右辺第3～4項の下添字Lは地形区分を、下添字Sは表層地質の岩石区分を表しており、各項の係数の分布は図9における結果と調和的である。図11に、説明変数の取り方と相関係数との間の関係を示しておく。ちなみに、(4)式による相関係数は各々0.845、0.843であり、加速度および速度増幅度の目標値（被害想定結果から求めた地盤増幅度の計算値）と推定値との関係は図12に示す散布図の通りである。参考のために、被害想定に用いた代表地盤モデルを用いて地盤の卓越周期および平均S波速度を評価し、(1)および(2)式⁵⁾～⁶⁾に基づいて速度増幅度を求めた結果を図13に示す。(1)式は増幅度を大きめに、(2)式は小さめに評価する傾向があることが分かる。(4)式を用いて東海地域全体の地盤増幅度分布を求めた結果を図14に示す。沖積平野部の速度増幅度をやや過小評価しているものの図3と比較的良い対応を示しており、(4)式は簡易的な増幅度評価法として利用できるものと考えられる。

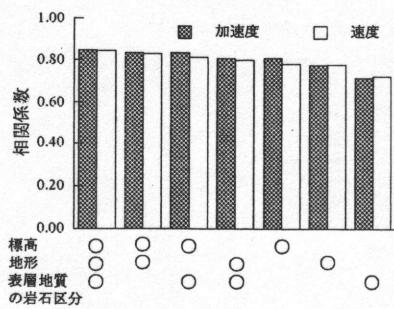


図 11 説明変数の取り方と相関係数

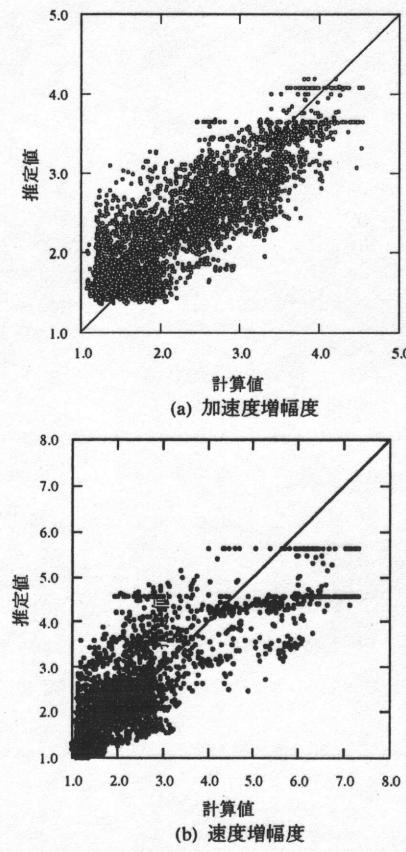


図 12 地盤増幅度の計算値と推定値の散布図

表 1 地形区分の再分類

| コード | 国土数値情報 | 本研究 |
|-------|---------|---------|
| 0 1 0 | 大起伏山地 | 山地 |
| 0 2 0 | 中起伏山地 | |
| 0 3 0 | 小起伏山地 | |
| 0 4 0 | 山麓地 | |
| 0 6 0 | 中起伏火山地 | (不採用) |
| 0 9 0 | 大起伏丘陵地 | 大起伏丘陵地 |
| 1 0 0 | 小起伏丘陵地 | 小起伏丘陵地 |
| 1 6 0 | 砂礫台地・段丘 | 砂礫台地・段丘 |
| 1 9 0 | 扇状地性低地 | 扇状地性低地 |
| 2 0 0 | 三角州性低地 | 三角州性低地 |
| 2 1 0 | 干拓地 | 干拓地 |
| 0 0 7 | 干拓地 | |
| 2 2 0 | 自然堤防・砂州 | 自然堤防・砂州 |
| 2 4 0 | 湖沼 | (不採用) |
| 2 5 0 | 河川 | 河川 |
| 0 0 6 | 埋立地・盛土 | 埋立地・盛土 |

表 2 表層地質の岩石区分の再分類

| コード | 国土数値情報 | 本研究 |
|-----|--------------------|-----|
| 1 1 | 未固結堆積物・礫層を主とする地域 | 礫 |
| 1 2 | 未固結堆積物・砂層を主とする地域 | 砂 |
| 1 3 | 未固結堆積物・泥層を主とする地域 | 泥 |
| 1 8 | 未固結堆積物・上記互層を主とする地域 | 互層 |
| 3 0 | 番台・固結堆積物 | 固結 |
| 5 0 | 番台・火山性岩石 | |
| 6 0 | 番台・深成岩類 | |
| 7 0 | 番台・変成岩類 | |

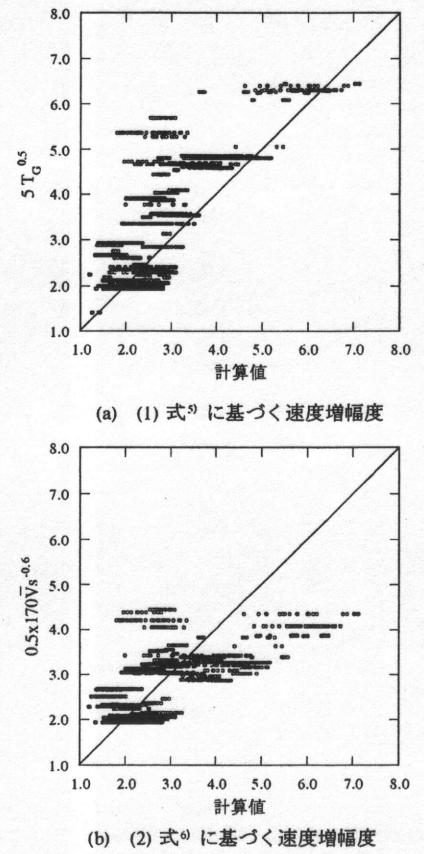
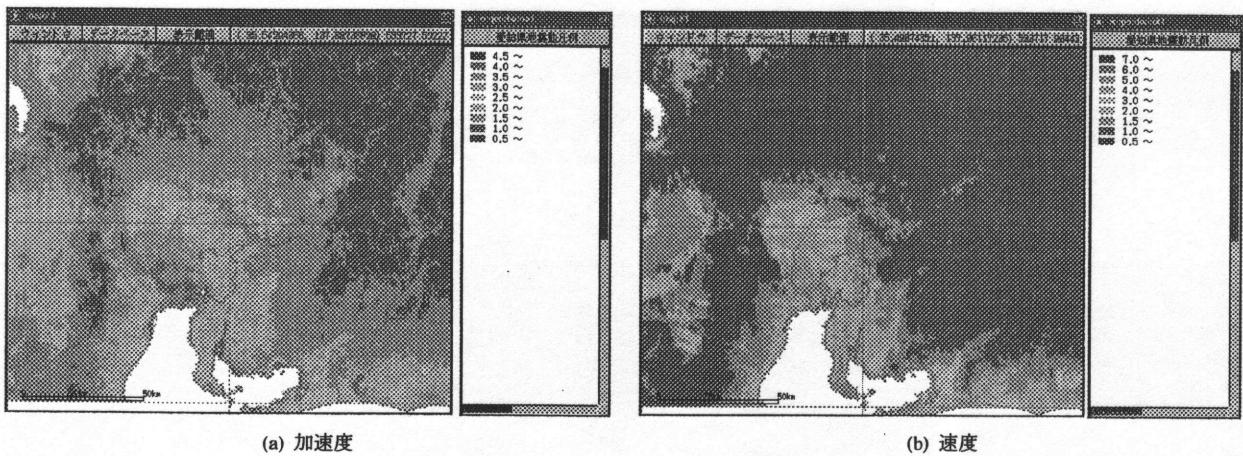


図 13 速度増幅度の計算値と既往の推定値の散布図



(a) 加速度

(b) 速度

図14 (4)式により推定された東海地域の地盤増幅度分布

4. 地盤の材料非線形性の考慮

名古屋市地震被害想定結果に基づき、3つの想定地震に対して入射波最大加速度と加速度増幅度の関係を地形・表層地質の岩石区分ごとに調べた。一例として図15に地形区分が埋立地・盛土の場合を示す。図のように入射波加速度の増加と共に、加速度増幅度が減少しており、地盤の材料非線形性の影響が明らかに認められる。同様の分析を全ての地形・表層地質の岩石区分について実施した結果、地形が三角州性低地、干拓地、自然堤防・砂州、河川、埋立地・盛土で、かつ表層地質の岩石区分が泥や砂の場合に加速度増幅度の低下が認められ、線形時の増幅度が大きいほど増幅度の低下の度合が大きいことがわかった。ちなみに、速度増幅度についても同様の検討を行った結果を図16に示すが、入射波速度振幅に対する依存性は加速度に比較して小さい。

そこで、非線形性の著しい濃尾地震と、ほぼ線形応答を示している想定東海地震の加速度増幅度の差を、両地震の入射波最大加速度の差で除したものを加速度増幅度減少率 $dARA$ と定義し、この減少率と線形時（想定東海地震時）の加速度増幅度との関係について検討した。図17に埋立地・盛土の場合を示す。同様の分析を三角州性低地、干拓地、自然堤防・砂州についても行った結果、線形時の加速度増幅度が大きいほど減少率が大きく、地形や泥と砂による減少率の違いはあまりみられなかった。

そこで、この関係を増幅度2.5以上のものについて直線回帰し(5)式を得た（図17参照）。

$$dARA = 0.025 - 0.01 ARA_0 \quad (5)$$

(5)式で得られる加速度増幅度減少率の推定値と被害想定結果に基づく計算値との散布図を図18に示すが、相関係数は0.749であり、地盤の材料非線形性に伴う加速度増幅

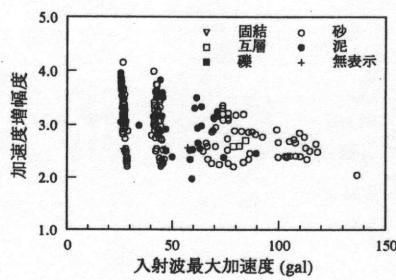


図15 埋立地・盛土の入射波最大加速度と加速度増幅度

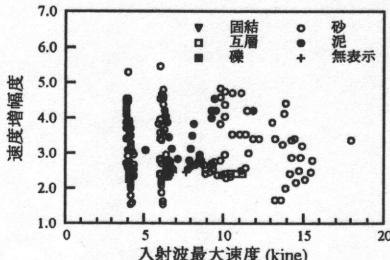


図16 埋立地・盛土の入射波最大速度と速度増幅度

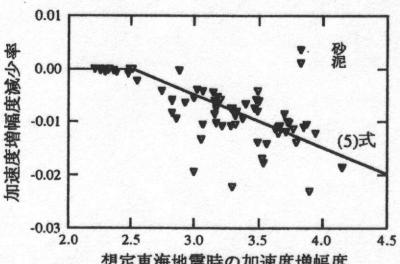


図17 埋立地・盛土の加速度増幅度減少率

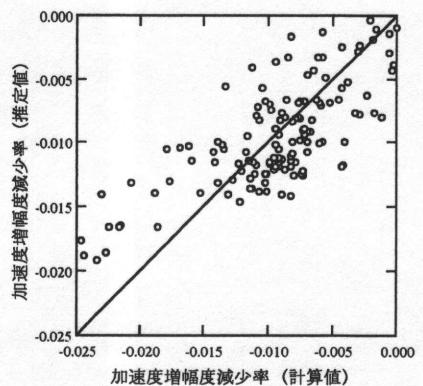


図18 加速度増幅度減少率の計算値と推定値の散布図

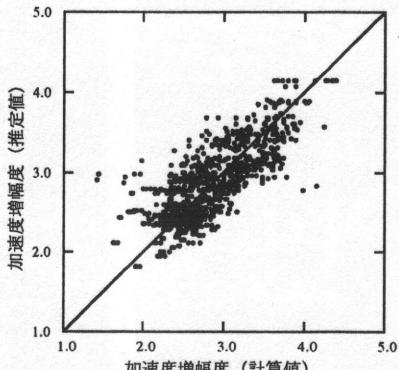


図19 非線形性を考慮した加速度増幅度の計算値と推定値の散布図

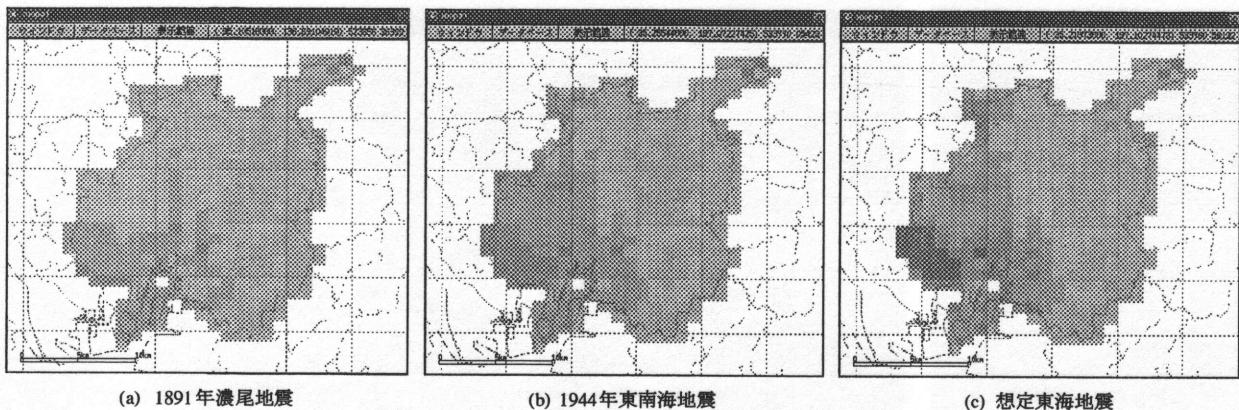


図20 地盤の材料非線形性を考慮した推定加速度増幅度

度の低減効果を概ね表現することができている。結果として、非線形性を考慮した加速度増幅度 ARA は、

$$ARA = ARA_0 + dARA \times (A_{max} - 30) \quad (6)$$

により得られる。ただし、 A_{max} は地震基盤における入射波最大加速度 (Gal)、右辺第2項の30Galは名古屋市域の地震基盤における想定東海地震時の平均入射加速度レベルである。また、上記4種類以外の地形および表層地質の岩石区分が泥と砂以外の場合と、増幅度が2.5未満のものについては増幅度を低減しないこととする。図19に(6)式で得られた加速度増幅度の推定値と名古屋市地震被害想定結果に基づく計算値との散布図を示す。また、図20に3つの想定地震に対する名古屋市内の加速度増幅度の推定値分布を示す。入力レベルの増加と共に名古屋市南西部の加速度増幅度が低下しており、図6に示した結果と良い対応を示している。なお、地震被害想定に等価線形化手法を用いている関係上、(6)式の適用範囲はせん断ひずみレベルで1%程度が上限の目安であると考えられる。

5.まとめ

本論では、地盤資料が十分に整っていない地域での地震時地盤挙動を予測するため、全国的に整備されている国土数値情報に基づいた地盤増幅度の推定式を提案し、その有効性を示した。ここでは、愛知県および名古屋市で実施された地震被害想定において得られた地震動予測結果を利用し、地表と地震基盤の最大振幅の比を地盤増幅度と定義して、国土数値情報の標高・地形・表層地質の岩石区分を説明変数とした重回帰分析を行い、国土数値情報から地盤増幅度を直接求める推定式を誘導した。また、速度増幅度と加速度増幅度の2つの推定式を作成し、加速度増幅度には地盤の材料非線形性の影響を考慮した。得られた結果は概ね満足できるものであり、全国的に整備されている国土数値情報を用いることによって、任意地点の地震動レベルの簡易予測を可能にすることことができた。ただし、目標値とした地震被害想定結果そのものも真値ではない点、地盤の成立過程の違いによって各

種地盤情報間の相関には地域性がある点、地震被害想定時に等価線形解析手法を用いているため推定式には適用限界が存在する点などには注意が必要であり、他地域への適用を考えていく際の今後の課題でもある。本論における提案式は、インテラクティブに地震動評価を行う地理情報システムや、即時性の要求されるリアルタイム地震被害予測などへの適用が考えられると共に、地盤増幅度を考慮した地震危険度解析などへの応用も考えられる^{14)~15)}。

謝辞

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A STUDY ON THE FREQUENCY AND DAMPING OF SOIL-STRUCTURE SYSTEMS USING A SIMPLIFIED MODEL

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A simplified 3DOF model is suggested as the soil-structure system by replacing the structure by a SDOF model using the modal coordinates of the fixed-base structure. The modal parameters of the system are evaluated parametrically through the eigenvalue analysis. The derived formulations allow the use of hysteretic damping model as the material damping in the soil and structure which is a more realistic model than the commonly used viscous one. The frequency dependency of the soil impedances can also be handled by the model through a few loops of iteration. The model is applied for the case of building on surface of half-space soil medium and satisfactory results are achieved in comparison to the results of more rigorous models and also code provisions. The model is also easily applicable for more complicated cases such as buildings on layered half-space soil media.

Key Words: Soil-Structure Interaction, Simplified Model, Complex Eigenvalue Analysis, Damping Ratio, Natural Frequency, Cone Models

Introduction:

It is well-known that the dynamic properties of structures are influenced by flexibility of soil under them due to Soil-Structure Interaction (SSI). As a result, the soil-structure system usually has a longer natural period and higher damping ratio than the structure would have in the fixed-base state. Especially, the latter effect is considerable due to the radiation damping in the soil and may significantly affect the structural response to dynamic loads. The importance of SSI effect as well as the complexity of the phenomenon have made it the subject of several researches for the last three decades.¹⁻¹⁹ It also has found its way into some seismic codes and provisions as simplified guidelines.^{20,21} Among different treatments with the subject, the possible application of modal analysis to interacting soil-structure system, with its inherent advantages, has attracted the attention of a number of researchers.⁴⁻¹¹ As a key

parameter in modal analysis, many researchers have put effort against estimating the modal damping coefficients. This has been mainly done by matching the rigorous and normal mode solutions of transfer function⁸ or by using energy methods⁵⁻⁷ which are based on the assumption of equal undamped and damped mode shapes. However, application of complex eigenvalue analysis is rare^{4,12,15} and almost limited to the case of structure on surface of soil half-space with frequency independent impedance functions. The complex eigenvalue analysis method was used by the authors¹⁶ for the case of structure located on surface of homogeneous half-space replaced by frequency dependent springs and dashpots using Cone Models.²² The method was then extended to the case of layer on flexible half-space.¹⁷ Here, as a further step, a simplified formulation based on a 3 degrees of freedom (DOF) replacement model is introduced which can predict the modal damping

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ratios of soil-structure systems with sufficient accuracy. There are also some other formulations based on well-established simplified models in the literature^{2,3} which are indeed the basis of current seismic code provisions. However, the present formulation is based on the complex eigenvalue analysis concept by using frequency dependent dynamic stiffness for soil. Moreover, it considers the effect of foundation's mass and also allows the hysteretic form of damping to be used as the material damping in both soil and structure which is a more realistic model than the commonly used viscous form of damping.^{23,24} The proposed model is able to capture the results of more rigorous model already used by the authors¹⁵⁻¹⁷ with much less effort. Moreover, it may be used for more general soil-structure systems.

The Basic Soil-Structure Model:

Figure 1 shows the conventional soil-structure model which has been used by several researchers. The structure is modeled as a shear building and the soil is replaced by sway and rocking springs and dashpots. It is common practice among researchers to use frequency independent coefficients for the soil representative springs and dashpots because of its simplicity.^{1,2,4,5,8} However, although the idea may practically work for the case of building on surface of homogeneous soil half-space, generally it is not applicable for the layered sites where the dynamic stiffness of soil varies drastically with frequency.¹⁹

Generally, the standard eigenvalue analysis is not applicable to the soil-structure system due to the

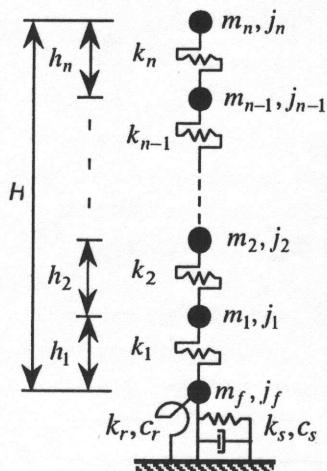


Figure 1. The conventional soil-structure model

difference in the nature of damping mechanism in the bounded structure and unbounded soil.⁷⁻⁹ It is because the damping matrix wouldn't be in a form to allow the system posses classical modes of vibration.^{25,26} Although there are well-established techniques for handling even non-classical damped systems,^{10,27} they are not applicable when the frequency dependency of soil stiffness is also considered. Also, the application of such techniques is limited to cases with real stiffness matrices where the use of complex damping for modeling the material damping in the soil or structure is not applicable. On the other hand, the results of experimental studies on structures are more compatible with the concept of the hysteretic damping model (complex damping) than the commonly used viscous form of damping.²³ Also, the hysteretic damping model has been proposed as the best possibility for modeling the material damping in the soil.²⁴ As an alternative method capable to deal with all above mentioned problems, explicit presentation of determinant of the stiffness matrix was used by the authors.¹⁵⁻¹⁷ The case of building with the same mass and stiffness for all stories was studied and the determinant of the stiffness matrix of the system was expressed by polynomials explicitly through the cofactor expansion method. However, special techniques are required to solve the equations in the complex plane. Additionally, the method is limited to the case of buildings with uniform distribution of mass and stiffness in height.

The Replacement 3DOF Model and Formulations:

The model of Fig.1 discussed in the previous section is replaced by a much simpler 3DOF model in this section. Figure 2 shows the simplified model where the superstructure -building- is replaced by its modal effective mass, $m_{str.}$, and modal effective stiffness, $k_{str.}$, providing the same modal frequency as the original multi degree of freedom (MDOF) model, as shown in Fig.1, in the fixed base state.

$$\omega_{fix} = \sqrt{k_{str.}/m_{str.}} \quad (1)$$

The effective mass for the q th mode is defined as

$$m_{str.} = \frac{\left[\sum_{p=1}^n m_p \varphi_{pq} \right]^2}{\sum_{p=1}^n m_p \varphi_{pq}^2} \quad (2)$$

where m_p is the mass of the p th story and φ_{pq} is the

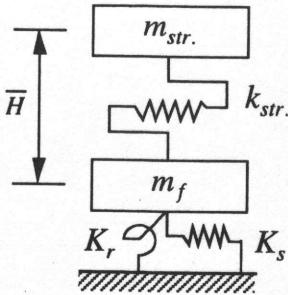


Figure 2. The simplified model

amplitude at p th story in the q th mode of vibration computed for the fixed-base MDOF model. Also, the q th mode's effective height, \bar{H} , is defined as

$$\bar{H} = \frac{\sum_{p=1}^n m_p \varphi_{pq} \left(\sum_{i=1}^p h_i \right)}{\sum_{p=1}^n m_p \varphi_{pq}} \quad (3)$$

The ratio of the modal effective mass and height factors, as defined in (2) and (3), to the total mass and height of the structure

$$m_{tot.} = \sum_{i=1}^n m_i \quad (4a)$$

$$H = \sum_{i=1}^n h_i \quad (4b)$$

are referred to as the effective mass and height ratios in this text and are shown by λ_m and λ_H , respectively.

$$\lambda_m = \frac{m_{str.}}{m_{tot.}}, \quad \lambda_H = \frac{\bar{H}}{H} \quad (5)$$

The foundation is represented by mass m_f in the simplified model and the springs K_s and K_r are considered as the frequency dependent dynamic stiffness of soil in the sway and rocking DOF, respectively. It should be emphasized that each of K_s and K_r has been replaced for both of the related spring and dashpot in the model of Fig.1 and consequently has complex value stiffness as follows:

$$K_s = k_s + i\omega c_s \quad (6a)$$

$$K_r = k_r + i\omega c_r \quad (6b)$$

where ω is the circular frequency of vibration and $i = \sqrt{-1}$.

Introducing the dimensionless parameters

$$\alpha = \frac{m_f}{m_{str.}}, \quad \beta = \frac{k_{str.}}{K_s}, \quad \gamma = \frac{K_r}{k_{str.} \cdot \bar{H}^2} \quad (7)$$

the mass and stiffness matrices of the simplified model can be written as follows.

$$M = m_{str.} \begin{bmatrix} 1 & 0 & \bar{H} \\ 0 & \alpha & 0 \\ \bar{H} & 0 & \bar{H}^2 \end{bmatrix} \quad (8a)$$

$$K = K_s \begin{bmatrix} \beta & -\beta & 0 \\ -\beta & 1+\beta & 0 \\ 0 & 0 & \beta \gamma \bar{H}^2 \end{bmatrix} \quad (8b)$$

The first complex eigenfrequency of the system then will be

$$\omega = \omega_{fix} \sqrt{\frac{2\gamma}{\zeta + \sqrt{\zeta^2 - 4\alpha\beta\gamma(1+\gamma)}}} \quad (9a)$$

where

$$\zeta = 1 + \gamma + \beta\gamma + \alpha\beta\gamma \quad (9b)$$

Using Cone Models, the coefficients of the soil springs and dashpots may be expressed as follows

$$k_s = K_H \cdot k_H, \quad c_s = K_H \cdot c_H \quad (10a)$$

$$k_r = K_\theta \cdot k_\theta, \quad c_r = K_\theta \cdot c_\theta \quad (10b)$$

where k_H , k_θ , c_H and c_θ are the frequency dependent dynamic coefficients and K_H and K_θ are the static stiffness of disk located on surface of a homogeneous half-space for the sway and rocking DOF, respectively, as follows

$$K_H = \frac{8\rho V_s^2 r}{2-\nu} \quad (11a)$$

$$K_\theta = \frac{8\rho V_s^2 r^3}{3(1-\nu)} \quad (11b)$$

in which ρ , V_s and ν are respectively the specific mass, shear wave velocity and Poisson's ratio of soil and r is the radius of the circular foundation. The equivalent radius may be used in the case of rectangular foundations by matching the area or moment of inertia of the foundation with a circular foundation replacement for the sway or rocking DOF, respectively.²² Using (1), (6) and (10), parameters β and γ in (7) are written in the following form

$$\beta = m_{str.} \omega_{fix}^2 / K_H (k_H + i\omega c_H) \quad (12a)$$

$$\gamma = K_\theta (k_\theta + i\omega c_\theta) / \bar{H}^2 m_{str.} \omega_{fix}^2 \quad (12b)$$

Also, by introducing two other dimensionless parameters as

$$\bar{m} = m_{tot.} / \rho r^2 H \quad (13a)$$

$$(a_0)_{fix} = r \omega_{fix} / V_s \quad (13b)$$

finally, (12) may be rewritten as follows

$$\beta = \left[\frac{2-\nu}{8} \bar{m} \lambda_m \frac{H}{r} (a_0)_{fix}^2 \right] (k_H + i\omega c_H)^{-1} \quad (14a)$$

$$\gamma = \left[\frac{3(1-\nu)}{8} \bar{m} \lambda_m \lambda_H^2 \left(\frac{H}{r} \right)^3 (a_0)_{fix}^2 \right]^{-1} (k_\theta + i\omega c_\theta) \quad (14b)$$

The eigenvalue of the system can then be evaluated by using (9) and (14) by iteration while the dynamic stiffness coefficients of k_H , c_H , k_θ and c_θ are updated in each loop. Then, the damped frequency and damping ratio of the system will be evaluated as follows

$$\omega_d = \text{Real}(\omega) \quad (15a)$$

$$\xi = \frac{\text{Imag}(\omega)}{\text{Abs}(\omega)} \quad (15b)$$

where $\text{Real}()$, $\text{Imag}()$ and $\text{Abs}()$ mean the real, imaginary and absolute values, respectively. The method is efficient and generally just a few iterations are required for the convergence into the result. At the end, it should be added that the material damping in the soil and structure may be included in the formulations as the hysteretic form of damping by using the correspondence principle,^{22,23} i.e., just by replacing the stiffness of the structure and soil by

$$\hat{k}_{str.} = k_{str.} (1 + 2\xi_{str.} i) \quad (16a)$$

$$\hat{K}_s = K_s (1 + 2\xi_{soil} i) \quad (16b)$$

$$\hat{K}_r = K_r (1 + 2\xi_{soil} i) \quad (16c)$$

where $\xi_{str.}$ and ξ_{soil} are the material damping ratios in the structure and soil, respectively.

Application of the Model to the First Mode of Vibration:

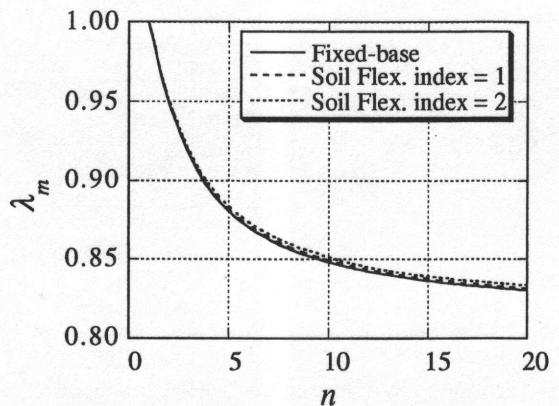
Generally, the first mode of vibration is the most important mode in the modal analysis of ordinary buildings. Also, the effect of SSI on the response of buildings may be taken into account with sufficient accuracy by considering only the change in the first mode's frequency and damping ratio due to SSI.^{20,21} Moreover, it was also shown by the authors^{16,28} that generally the higher modes of the soil-structure system have lower damping ratios due to lower interaction effect. The exception for this finding are the cases of flexible structures located on relatively stiff soil where SSI doesn't play any important role. Thus, the effect of SSI on the characteristics of the first mode of vibration of the soil-structure system is studied here using the introduced simplified model.

The concept of the model and formulations are also applicable for the higher modes of vibration by some minor modifications. However, they are not discussed herein.

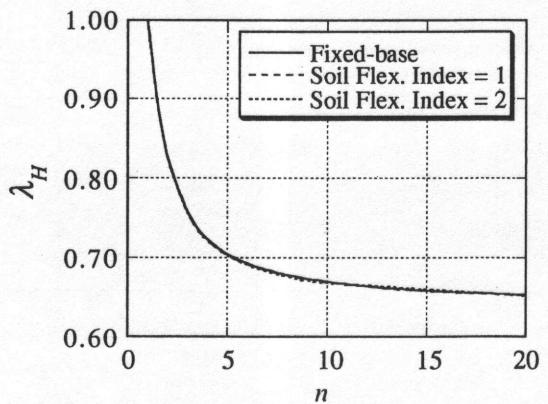
Before discussing the results, a point regarding buildings with low aspect ratios is noteworthy. Since the stories' mass moment of inertia have been neglected in the simplified model, it may cause an error in the results for such squat buildings in which the sum of the mass moment of inertia of stories is comparable or higher than the value of $m_{str.} \bar{H}^2$. This problem will be resolved with sufficient accuracy by replacing λ_H with $\hat{\lambda}_H$ as defined below.

$$\hat{\lambda}_H = \lambda_H \sqrt{1 + \frac{1}{4\lambda_H^2 (H/r)^2}} \quad (17)$$

This modification factor approaches to unity for higher aspect ratios very rapidly and doesn't introduce any specific error for the case of tall buildings.



(a) The effective mass ratio



(b) The effective height ratio

Figure 3. The effective mass and height ratios for different structure-soil stiffness ratios.
(Soil Flexibility Index = $(a_0)_{fix} \cdot H/r$)

The non-dimensional parameters used in the formulation of the simplified model may be summarized as

$$(a_0)_{fix}, H/r, \bar{m}, \alpha, \lambda_m, \lambda_H \quad (18)$$

in addition to four soil dynamic stiffness coefficients which are frequency dependent and updated in each loop of iteration using Cone Modes. Among the parameters in (18), the first two parameters have been selected as the key parameters here. The third one may be set to a typical value of $\bar{m} = 0.5$ for ordinary buildings. The other three parameters can easily be evaluated for any specific case when the mass, stiffness and height of each story as well as the mass of the foundation are known. As an example, a building with the same mass, stiffness and height for all stories attached to a foundation whose mass is the same as stories is examined here. This is the same model as the one used by the authors in their previous studies.¹⁵⁻¹⁷ Therefore, the results may be compared to those of the studies for evaluation of the proposed model. The variation of the last two parameters in (18) with the number of stories for such a case are shown in Fig.3. Since the existence of the soil affects the mode shapes of the structure and consequently the effective mass and height ratios, their variation should be taken into account in order to secure the reliability of the results. In this regard, the results for two levels of structure-soil stiffness ratio are also presented in Fig.3 in comparison with the results of the fixed-base model. As seen, the effect of soil flexibility on the mass and height effective ratios of the structure are negligible and the values related to the fixed-base model may be used with sufficient

accuracy. The remaining parameter in (18), α , will also be evaluated as follows for such an example with the same mass for all stories and the foundation.

$$\alpha = 1/(n \lambda_m) \quad (19)$$

Equation (19) and Fig. 3 reveal that the last three parameters in (18) are functions of the number of stories in the MDOF model. This may weaken the general applicability of the model. It is, however, shown here that despite the dependency of these parameters on the number of stories, the final results of the simplified model will be independent of this factor. It means that different sets of parameters α , λ_m and λ_H related to structural models with different number of stories lead to the same results for the change in the natural period and for the damping ratio of the system. However, for this purpose, instead of the aspect ratio of the building, the effective aspect ratio defined as

$$\frac{\bar{H}}{r} = \frac{H}{r} \cdot \lambda_H \quad (20)$$

should be used as the key parameter along with $(a_0)_{fix}$. Figure 4 shows the results for three different sets of parameters α , λ_m and λ_H corresponding to buildings with three different number of stories. The results are shown for a wide range of $(a_0)_{fix}$ covering the systems with no SSI, $(a_0)_{fix}=0$, to the systems with severe SSI effect with $(a_0)_{fix}=2.0$.²⁸ It should be noted that material damping is not considered in either the soil or the structure for the drawing shown in Fig.4. As seen in Fig.4, the results are essentially independent of the number of stories. This allows to set the last three parameters in (18) to some typical

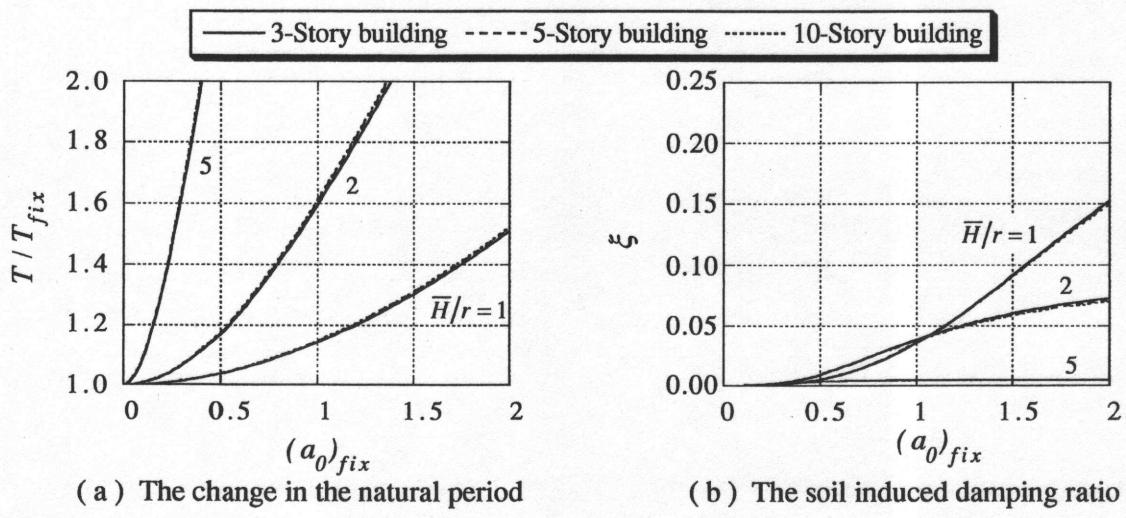
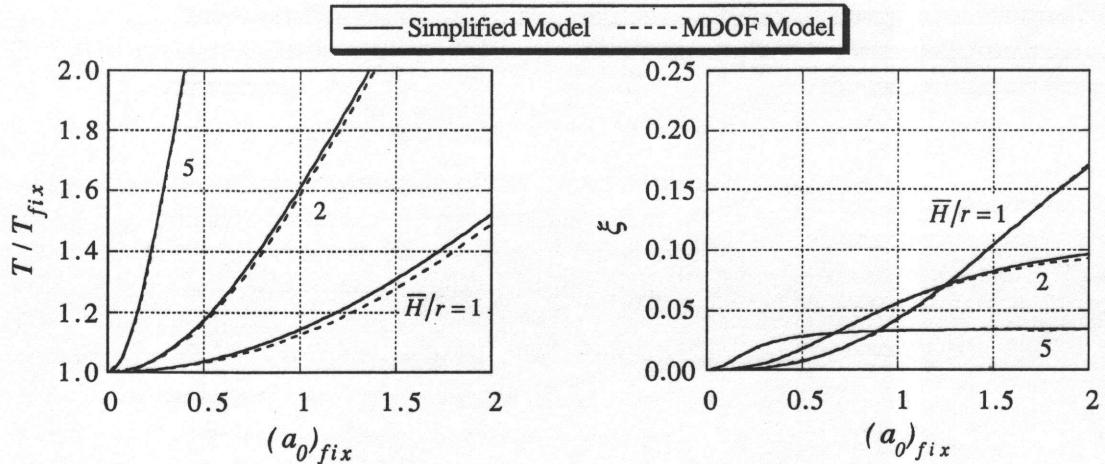


Figure 4. Comparison of the results for buildings with different number of stories
(No material damping is addressed in the soil and structure)



(a) The change in the natural period

(b) The soil induced damping ratio

Figure 5. Comparison of the results of the simplified and MDOF models ($\xi_{soil} = 0.03$)

constant values. Here the following values are used

$$\alpha = 0.118, \lambda_m = 0.848, \lambda_H = 0.669 \quad (21)$$

which belong to a 10-story building model.

Representative Results:

Using the values in (21), the results in the same manner as Fig.4 are drawn in Fig.5 in comparison with those for the MDOF model.²⁸ However, here a hysteretic material damping ratio of 3% has been considered in the soil. As shown, the results of the two models are almost identical, reinforcing the applicability of the simplified model. This model provides a more accurate approximation for the

natural period of the system than those suggested by ATC3-06²⁰ and NEHRP²¹. This is mainly because of disregarding the effect of the floors' mass moment of inertia in the model used by ATC3-06 and NEHRP which may lead to significant errors for short and squat buildings.²⁸ Since the regulations of ATC3-06 and NEHRP are almost the same, they just will be referred to as ATC3-06 hereafter in this text.

The results of Fig.5 may be presented in a new format as the variation of the system's damping ratio with the change in the natural period of the system by omitting $(a_0)_{fix}$ between parts (a) and (b) of the figure. This format is generally more desirable from the practical structural design point of view. Figure 6 shows the results of this study in this new format

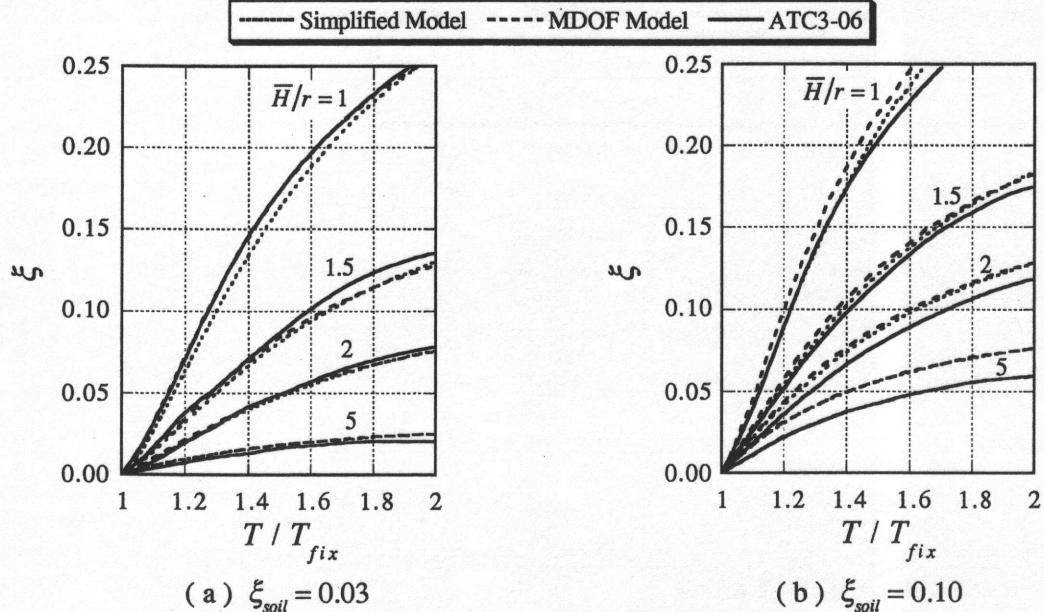


Figure 6. Comparison of the simplified model results with the MDOF and seismic code provisions

for two levels of material damping in the soil. Also drawn in the figure are the results of the MDOF model²⁸ and the graphs suggested by ATC3-06. It should be mentioned that the values of 3 and 10 percent as the material damping in the soil have been selected consistent with two soil strain levels introduced by ATC3-06. Empirical relationships have been used in this regard.^{29,30} The results of the simplified model in Fig.6 show full agreement with those of the MDOF model which are in turn in good agreement with ATC3-06 results. In fact, the simplified model and the MDOF model practically lead to the same results and for some cases, including tall buildings, the results of two models are even not distinguishable from each other in Fig.6. Also, the gap between the results of these two models with those of ATC3-06 can well be explained by the empirical nature of the used relationships between the expected material damping and the strain level in the soil. This confirms the reliability of the proposed model and its applicability. The model also has the capability to be used for more complicated cases such as buildings located on layered half-space soil media,²⁸ for which no detailed information is currently available.

For completing the discussion, the effect of soil-structure interaction on the structural damping itself should also be addressed. As the soil flexibility changes the vibration modes of the structure, the amount of energy dissipated in the structure itself also changes.¹⁴ This leads to lower internal (material) damping in the structure for the soil-structure system due to reduction of curvature. Although in lightly damped structures this less desirable effect would be

negligible, for the case of structures with heavy internal damping, the effect may be considerable.¹² Also, it has been pointed out that in the cases of structures with large internal damping, the loss of damping due to this effect may be greater than the gain due to radiation damping in the soil.^{4,12} This phenomenon is more probable with slender structures.³ However, it has been shown that this possibility doesn't occur when structural damping is assumed to be hysteretic rather than viscous.¹⁸ The variation of structural damping due to soil flexibility is also studied here. The structural damping reduction factor, δ , is defined as follows in this regard.

$$\delta = \frac{\xi_{str.}}{\xi_{str.}} \quad (22)$$

where $\xi_{str.}$ and $\xi_{str.}$ are the internal damping ratio of the structure in the fixed-base state and when located on flexible soil, respectively. The variation of δ has been approximated by some researchers as follows^{13,29}

$$\delta = \left(\frac{T_{fix}}{T} \right)^3 \quad (23)$$

which is valid for systems with viscous type of material damping in the structure. The same approximation has been also used by ATC3-06. The assumption of hysteretic type of damping for the structure, as adopted here, however, leads to an exponent 2 (instead of 3) in the right-hand side of (23)¹⁹, i.e.,

$$\delta = \left(\frac{T_{fix}}{T} \right)^2 \quad (24)$$

Figure 7 shows the results of this study for systems

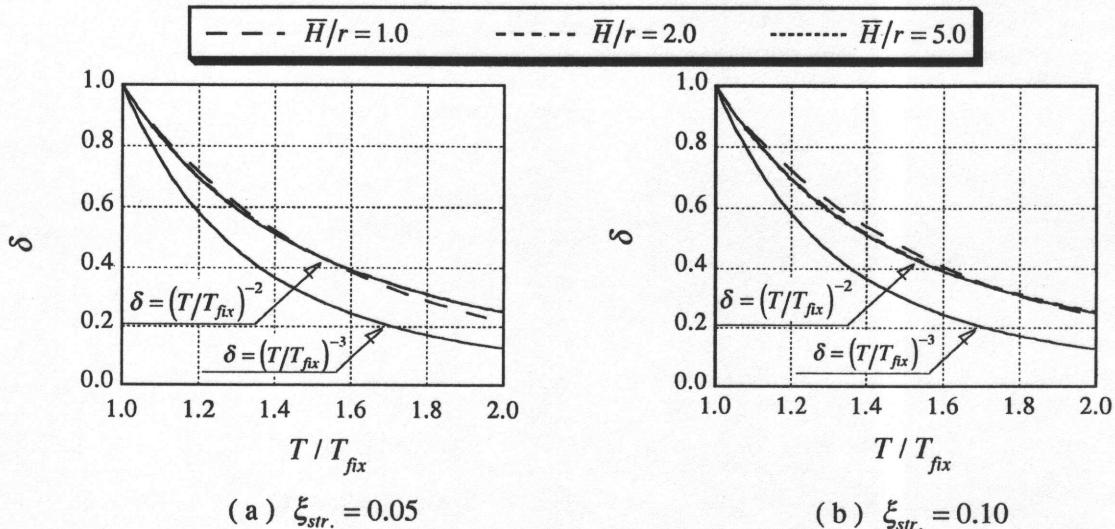


Figure 7. The effect of soil flexibility on the internal damping of the structure

with two different levels of internal damping and with different values of effective aspect ratios for the structure. Also drawn in this figure are the curves of Eqs. (23) and (24). As shown, the results of this study are well matched with Eq.(24) as expected for a system with hysteretic material damping model. Especially for slender buildings the results almost coincide the curve of Eq.(24) and they are not distinguishable from each others.

Conclusion:

A simple 3DOF model was introduced as the replacement for more complicated soil-structure systems. The dynamic properties of the model for the first mode of vibration were calculated through the parametric complex eigenvalue analysis. Code type graphs for the relationship between the soil induced damping ratios and the change in the period of building were drawn which are in good agreement with the regulations of ATC3-06 and NEHRP. Although the model doesn't consider for the number of stories, the results are in full agreement with those of the MDOF building-soil models with different number of stories but the same effective aspect ratio of building. The model is also applicable to more sophisticated cases such as building on a layered half-space soil medium, for which no detailed information is currently available.

Nomenclature:

$(a_0)_{fix}$: Dimensionless frequency computed for the fixed base state of the structure
 c_r : Soil representative dashpot for the rocking DOF
 c_s : Soil representative dashpot for the sway DOF
 c_H : Dimensionless damper coefficient for the sway DOF
 c_θ : Dimensionless damper coefficient for the rocking DOF
 h_i : Height of the i th story
 H : Total height of the structure
 \bar{H} : Effective height of the structure, (Eq.3)
 j_p : Mass moment of inertia of the p th story
 j_f : Mass moment of inertia of the foundation
 k_i : Total stiffness of the i th story
 k_r : Soil representative spring for the rocking DOF
 k_s : Soil representative spring for the sway DOF
 $k_{str.}$: Modal effective stiffness of the structure
 k_H : Dimensionless spring coefficient for the sway DOF

k_θ : Dimensionless spring coefficient for the rocking DOF
 K : Stiffness matrix
 K_H : Static stiffness of disk on surface of soil half-space in the sway DOF
 K_θ : Static stiffness of disk on surface of soil half-space in the rocking DOF
 K_s : Generalized dynamic stiffness of the soil in the sway DOF (Eq.6a)
 K_r : Generalized dynamic stiffness of the soil in the rocking DOF (Eq.6b)
 $\hat{k}_{str.}$: Complex modal stiffness of the structure (Eq.16a)
 \hat{K}_s : Complex dynamic stiffness of the soil in the sway DOF (Eq.16b)
 \hat{K}_r : Complex dynamic stiffness of the soil in the rocking DOF (Eq.16c)
 m_f : Mass of the foundation
 m_p : Mass of the p th story
 $m_{str.}$: Modal effective mass of the structure (Eq.2)
 $m_{tot.}$: Total mass of the structure
 M : Mass matrix
 \bar{m} : Structure-soil mass ratio index (Eq.13a)
 n : Number of stories
 r : (Equivalent) Radius of circular foundation
 V_s : Shear wave velocity in soil

 α : Foundation-structure mass ratio (Eq.7)
 β : Structure-soil lateral stiffness ratio (Eq.7)
 γ : Structure-soil rotational stiffness ratio (Eq.7)
 ζ : Dimensionless parameter as defined in Eq.9b
 λ_m : Effective mass ratio of the structure (Eq.5)
 λ_H : Effective height ratio of the structure (Eq.5)
 $\hat{\lambda}_H$: Modified effective height ratio (Eq.17)
 δ : Structural damping reduction factor (Eq.22)
 ν : Poisson's ratio in the soil
 φ_{pq} : Amplitude at the p th story in the q th mode of vibration
 ρ : Soil mass density
 ω : Complex eigenfrequency of the soil-structure system
 ω_{fix} : Fundamental circular frequency of the fixed-base structure
 ω_d : Damped frequency of the soil-structure system
 ξ : Damping ratio of the soil-structure system
 $\xi_{str.}$: Internal (material) damping ratio in the structure at the fixed-base state
 $\bar{\xi}_{str.}$: Internal (material) damping ratio in the structure when located on flexible soil

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